Modeling Hubbert curves as MIP formulation in TIMES

Uwe Remme

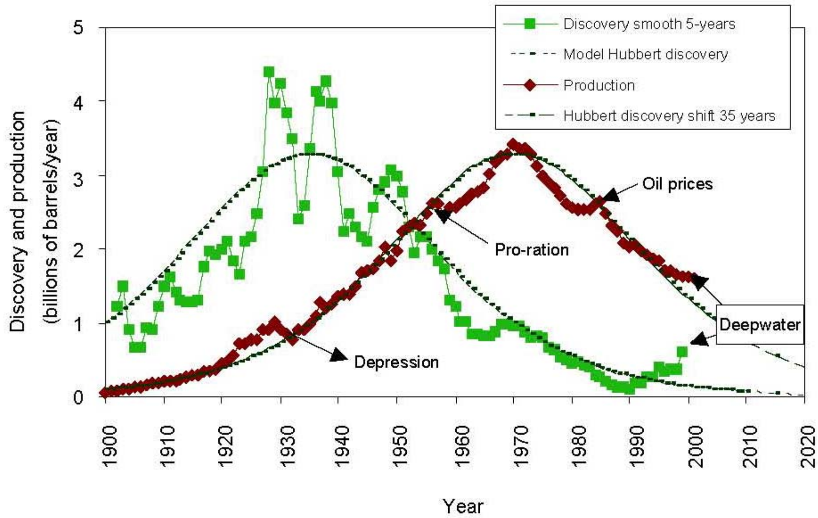
Institute of Energy Economics and the Rational Use of Energy (IER)
Energy Economics and System Analysis (ESA)

MARKAL-TIMES: Understanding ETSAP tools
July 2nd, 2008 Paris

Overview

- Introduction to Hubbert curves
- Approximation of Hubbert curves using Mixed Integer Programming:
  i. Logistic Hubbert curves
  ii. Big-M formulation
  iii. Convex hull formulation
- Preliminary example results

Hubbert curve (USA, 48 states)



Source: Laherrère, 2003

Hubbert curves

„Discoveries self-regulate discoveries"

$$\frac{\partial CD(t)}{\partial t} \propto CD(t) \cdot \left(Q_\infty - CD(t)\right)$$

Learning effect — Depletion effect

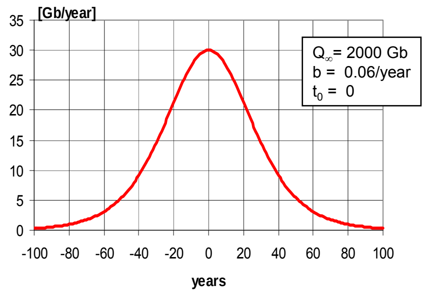
$CD(t)$: Cumulative discoveries

Q_∞ : Oil-in-place

Discoveries follow a Hubbert curve

Production follows a Hubbert curve

$$x_{t,k} = \frac{URR_k \cdot b_k \cdot e^{-b_k (t - t_{0,k})}}{\left(1 + e^{-b_k (t - t_{0,k})}\right)^2}$$



$x_{t,k}$: Production of category k

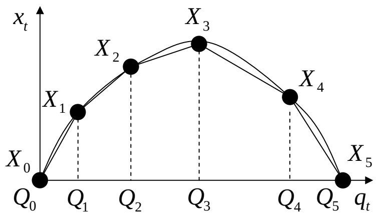
URR_k : Ultimate recoverable resource category k

b_k : Steepness parameter k

$t_{0,k}$: Peak year

Logistic Hubbert curves as MIP

$$x_t = q_t \cdot b \cdot \left(1 - \frac{q_t}{URR}\right)$$ *Relationship for Hubbert curve*



Piece-wise linear approximation of non-convex function

$$x_t = \lambda_{0,t}X_0 + \lambda_{1,t}X_1 + \lambda_{2,t}X_2 + \lambda_{3,t}X_3 + \lambda_{4,t}X_4 + \lambda_{5,t}X_5$$

$$q_t = \lambda_{0,t}Q_0 + \lambda_{1,t}Q_1 + \lambda_{2,t}Q_2 + \lambda_{3,t}Q_3 + \lambda_{4,t}Q_4 + \lambda_{5,t}Q_5$$

$$\lambda_{0,t} + \lambda_{1,t} + \lambda_{2,t} + \lambda_{3,t} + \lambda_{4,t} + \lambda_{5,t} = 1$$

Piece-wise linear approximation

$$q_t = \sum_{\tau=1}^{t} x_\tau$$

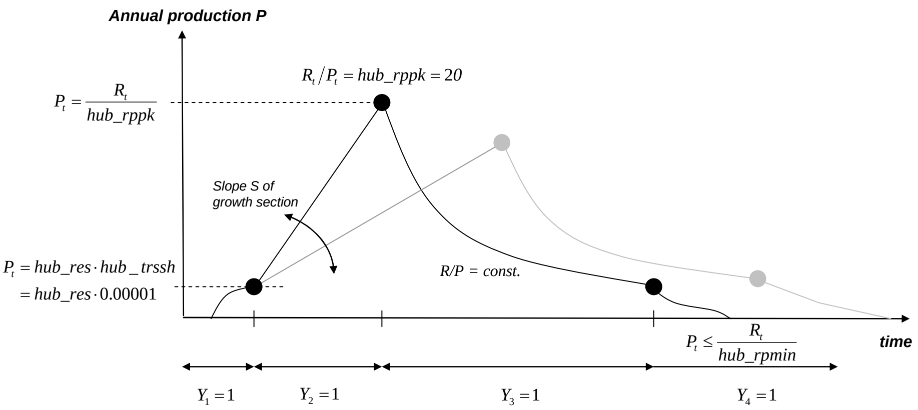
Ensuring only two neighboring points are selected

$$\begin{aligned}
\lambda_{0,t} &\leq y_{0,t} \\
\lambda_{1,t} &\leq y_{0,t} + y_{1,t} \\
\lambda_{2,t} &\leq y_{1,t} + y_{2,t} \\
\lambda_{3,t} &\leq y_{2,t} + y_{3,t} \\
\lambda_{4,t} &\leq y_{3,t} + y_{4,t} \\
\lambda_{5,t} &\leq y_{4,t}
\end{aligned}$$

$$y_{0,t} + y_{1,t} + y_{2,t} + y_{3,t} + y_{4,t} + y_{5,t} = 1$$

x_t	Annual production in t
q_t	Cumulative production until t
$\lambda_{i,t}$	Weighting factor of point i of approximated curve valid in period t
$y_{i,t}$	Binary variable to ensure only neighboring points are chosen
i	Set of points
t	Time periods
X_i, Q_i	Selected points used to approximate Hubbert curve

Simplified approach for production profiles



- Four sections of curve; Peak specified by minimum resource to production ratio
- Piece-wise linear sections described as disjunctive constraints using Mixed-Integer Programming

Approximation: Big-M method

Disjunctive constraints

$$A_1 x \le b_1 \quad \vee \quad A_2 x \le b_2 \quad \vee \quad A_3 x \le b_3 \quad \vee \quad A_4 x \le b_4$$

„Big M“ Formulation

$$\begin{aligned} A_1 x_1 &\le b_1 + M_1 \cdot (1 - y_1) \\ A_2 x_2 &\le b_2 + M_2 \cdot (1 - y_2) \\ A_3 x_3 &\le b_3 + M_3 \cdot (1 - y_3) \\ A_4 x_4 &\le b_4 + M_4 \cdot (1 - y_4) \\ y_1 + y_2 + y_3 + y_4 &= 1 \\ y_1, y_2, y_3, y_4 &\in 0,1 \\ x &\ge 0 \end{aligned}$$

Advantage: Few constraints

Disadvantage: Loose LP relaxation

Universität Stuttgart
Institut für Energiewirtschaft und Rationelle Energieanwendung IER

Approximation: Convex hull formulation

Disjunctive constraints

$$A_1 x \le b_1 \quad \vee \quad A_2 x \le b_2 \quad \vee \quad A_3 x \le b_3 \quad \vee \quad A_4 x \le b_4$$

„Convex hull“ Formulation

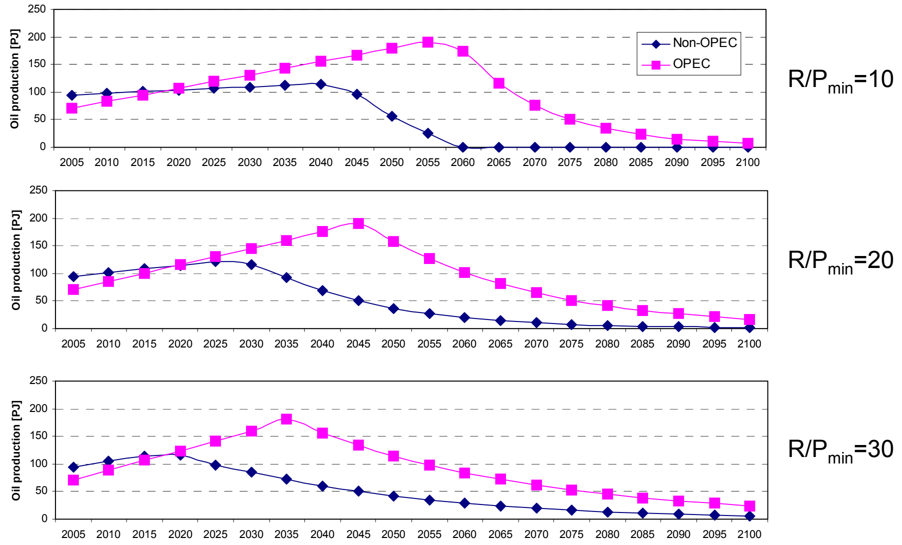
$$\begin{array}{cccc} A_1 z_1 \le b_1 y_1 & A_2 z_2 \le b_2 y_2 & A_3 z_3 \le b_3 y_3 & A_4 z_4 \le b_4 y_4 \\ 0 \le z_1 \le U_1 y_1 & 0 \le z_2 \le U_2 y_2 & 0 \le z_3 \le U_3 y_3 & 0 \le z_4 \le U_4 y_4 \end{array}$$

$$\begin{aligned} x &= z_1 + z_2 + z_3 + z_4 \\ y_1 + y_2 + y_3 + y_4 &= 1 \\ y_1, y_2, y_3, y_4 &\in 0,1 \\ z_1, z_2, z_3, z_4 &\ge 0 \\ x &\ge 0 \end{aligned}$$

Advantage: Tight LP relaxation

Disadvantage: More variables and constraints

Example: Hubbert curves in the TIAM model



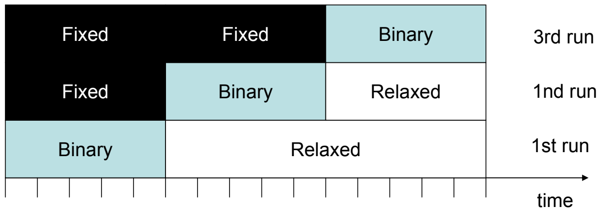
Universität Stuttgart
Institut für Energiewirtschaft und Rationelle Energieanwendung IER

Conclusions

- MIP representation of Hubbert curves as convex hull formulation operational in larger models.
- Critical factor in terms of computation time is the number of Hubbert curves,i.e., binary variables.
- Individual Hubbert curves for different regions and cost categories may require different types of approximations.

Notes on MIP in TIMES

- Block-wise capacity extension using a Rolling Horizon approach to solve larger models or more integer variables:



- Solver tolerances and options critical for results, to loose tolerances may lead to different solutions.