

Energy Technology Systems Analysis Programme

The Dual Solution of a TIMES Model: its interpretation and price formation equations

Authors:

**Uwe Remme; Markus Blesl; Tom Kober
IER, Universität Stuttgart**

Draft

July, 2009

Foreword

Table of contents

1. INTRODUCTION.....	5
2. DUAL SOLUTION OF A TIMES MODEL	6
2.1 Primal and Dual formulation of an LP Problem.....	6
2.1.1 Primal and dual problem	6
2.1.2 Key characteristics of duality	9
2.2 Dual Solution of TIMES Model.....	10
2.2.1 Activity definition EQ_ACTFLO.....	14
2.2.2 Activity bound EQ(l)_ACTBND	14
2.2.3 Activity-based efficiency relationship EQE_ACTEFF	14
2.2.4 Activity bound EQ(l)_BNDNET.....	15
2.2.5 Activity bound EQ(l)_BNDPRD.....	15
2.2.6 Capacity-Activity constraint EQ(l)_CAPACT.....	16
2.2.7 Commodity balance EQ(l)_COMBAL.....	16
2.2.8 Commodity production definition EQE_COMPRD	17
2.2.9 Capacity bound EQ(l)_CPT	17
2.2.10 Cumulative production/net consumption bound for a commodity EQ(l)_CUMPRD/NET	18
2.2.11 Bound on process flow EQ(l)_FLOBND	18
2.2.12 Bound on process flow EQ(l)_FLOFR.....	19
2.2.13 Bound on process flow EQ(l)_FLOMRK	20
2.2.14 Inter-regional exchange equation EQ_IRE	20
2.2.15 Bound on process flow EQ(l)_IREBND	21
2.2.16 Bound on process flow EQ(l)_XBND.....	21
2.2.17 Peaking equation EQ_PEAK.....	21
2.2.18 Transformation equation EQ_PTRANS.....	22
2.2.19 Input/output share constraint EQ(l)_IN/OUTSHR.....	22
2.2.20 Bound on storage flow EQ(l)_STGIN/OUT	23
2.2.21 Storage equation EQ_STGTSS/IPS	23
2.2.22 User constraints	23
2.2.23 Direct bounds on variables	25
3. PRICE FORMATION EQUATIONS	26
3.1 Price Formation Equations (dual constraints).....	26
3.2 Price Formation Equation of the Activity Variable	28
3.3 Price Formation Equation of the New Capacity Variable.....	32
3.4 Cost-Benefit Analysis	34
4. MATRIX TOOLS	37
4.1 Overview of the matrix tools	37
4.2 Attributes in VEDA-BE	39

5. SUMMARY AND OUTLOOK.....	43
6. REFERENCES	44

1. INTRODUCTION

The energy system model TIMES (Loulou et al. 2005) is a dynamic optimization model to analyze the energy sector in a technology rich fashion (so-called bottom-up modeling approach). In its standard formulation, a TIMES model is formulated as a so-called linear programming (LP) problem. It means that all the constraints to describe the energy system (e.g. efficiency relationship for an energy technology) as well as the objective function describing the goal of the optimization are of linear nature and that the decision variables, which are determined by the optimization (e.g. activity level of technologies or new capacity additions), are continuous and positive¹. The solution of an LP problem, and thus a TIMES model, consists of two parts: the primal and the dual solution. The primal solution of a TIMES model provides the optimal values for the decision variables of the original or primal problem. For example primal solution variables of a TIMES model are activity levels, energy flows or new capacity additions. The dual solution of a TIMES model provides additional information in terms of marginal or opportunity costs, which are assigned to each constraint of the primal problem. For example the dual solution associated with an upper CO₂ emission constraint describes the marginal costs accrued when in addition to the reduction given by the constraint a further ton of CO₂ would be mitigated. Hence, to achieve the given reduction target only those measures are taken by the model, which have mitigation costs equal or less the marginal costs of the constraints. In addition, one can transform the primal problem in an equivalent so-called dual problem with the dual variables being the decision variables. The constraints of this dual problem describe the relationship between the different dual variables. The analysis of these dual constraints is for example helpful to understand why a certain technology is or is not cost-competitive under the prevailing scenario conditions Overall, the dual solution of a TIMES model provides the modeler with valuable insights to deepen the understanding of the obtained solution.

The purpose of this report is to provide an overview of the dual solution available from a TIMES model and to demonstrate ways how this information can be used to enhance the understanding of the obtained model solution and thus the model itself. The first part of the report describes how the dual solution associated with the different primal constraint types of a TIMES model can be interpreted, while the second part of the report presents the dual constraints and how these constraints can be used to assess the cost-competitiveness of technologies. To give the modeler direct access to the primal and dual constraints a tool has been developed that extracts the constraint matrix of a TIMES model plus its primal and dual solution and processes these data in such a way that the primal and dual constraints and related solution attributes can be accessed and viewed in the data-cube software VEDA-BE. The last part of this report gives an overview of the implementation and usage of this tool to access the different problem matrices.

¹ Variants or extensions of TIMES exist which change the type of the optimization problem. So, results the restriction of the new capacity additions to certain block sizes or the introduction of learning curves in TIMES to a so-called mixed integer programming problem (MIP), i.e., some of the decision variables are not continuous anymore, but are restricted to integer or binary values. The MACRO extension, i.e., the linkage of a small economic model (computable general equilibrium model) with the energy system model, leads due to a non-linear objective function to a so-called non-linear programming problem (NLP).

variables (production level of a technology), input and output flow variables of technologies and the new capacity variables. The formulation of the LP problem shown in eqn. (1) is referred to as primal problem and its optimal solution as primal solution of the problem.

The so-called dual problem of the primal problem (1) is shown in eqn. (2). One obtains the constraint row j of the dual problem by transposing the column j of the primal problem.

$$\begin{aligned}
 \text{Max} \quad & b_1 y_1 + b_2 y_2 + \dots + b_m y_m & (2) \\
 & a_{11} y_1 + a_{12} y_2 + \dots + a_{1m} y_m \leq c_1 \\
 & a_{21} y_1 + a_{22} y_2 + \dots + a_{2m} y_m \leq c_2 \\
 & \quad \quad \quad \vdots + \\
 & a_{j1} y_1 + a_{j2} y_2 + \dots + a_{jm} y_m \leq c_j \\
 & \quad \quad \quad \vdots \\
 & a_{n1} y_1 + a_{n2} y_2 + \dots + a_{nm} y_m \leq c_n \\
 & \quad \quad \quad y_i \geq 0
 \end{aligned}$$

with

y_i = dual variable i .

The cost coefficients c_j of the primal problem become RHS constants in the dual problem. Vice-versa the RHS constants b_i of the primal problem are the objective function coefficients in the dual problem. For each constraint of the primal problem, there is a so-called dual variable y_i in the dual problem. Examples for dual variables are the prices of energy carriers linked to the energy balances in the primal problem or the marginal GHG abatement costs linked to an upper GHG bound in the primal problem. Simply said, each constraint of the primal problem is assigned through the dual variables an economic value or a price³. The value of the dual variable corresponds to the change in the optimal objective function value, if the RHS of the primal constraint is increased by one unit. Considering for example an upper GHG emission bound in the primal problem, the corresponding dual variable represents the value or the price for a certificate to emit one unit of GHG emissions. In such a way, each dual variable can be given one (or several) economic interpretations as discussed in detail in section 2.2. The dual constraints establish a relationship between the different dual variables and the cost coefficients of the primal problem. The dual constraints can be used to assess the cost-effectiveness of a technology as presented more elaborate in section 3. The optimal solution value of the primal and the dual problem are identical, which is one characteristic of the duality theorem outlined in the next section 2.1.2.

If the primal constraint consists only of one variable with a coefficient (so-called row singleton), the constraint represents an upper, lower or fixed bound on the variable depending on the constraint type. The corresponding dual variable is also referred to as *reduced costs*. They describe the change in the primal objective function, if the lower or upper bound is increased by one unit (a fixed bound can be replaced by an upper and lower bound). The reduced costs are only non-zero, if the primal variable is at its upper or lower bound. Even, if no explicit lower bound is specified for a primal variable, the variable has a

³ This economic interpretation of the dual variable is only valid as long as the objective function of the primal problem measures economic quantities as costs, revenues etc.

lower bound of zero due to the non-negativity condition in (1). If the decision variable is at this lower bound, the corresponding dual variable can also be interpreted as the amount, by which the cost coefficient of the primal variable in the objective function has to be reduced in order to make the decision variable cost-effective in the primal problem. For that reason the dual variables corresponding to lower bounds on variables are called reduced costs.

The dual problem presented in (2) has been derived from a primal problem in a standard form (Von-Neumann symmetric form), i.e. all constraints are of the same inequality type. Alternatively, one can construct the dual problem from the original primal problem using the equivalences given in Table 1. It can be seen that the primal constraint type determines already the sign of the dual variable. For example, a lower bound primal constraint (\geq) corresponds with a positive (strictly speaking non-negative) dual variable. When this constraint is binding, i.e. the inequality is satisfied with equality at its lower bound, it indicates that in order to minimize the total system costs the optimization routine would have reduced the term on the LHS further beyond the given lower bound, since this contributes to the minimization of the objective function. Hence, an increase of this lower bound yields also an increase in the objective function, which explains the positive sign of the dual variable for the lower bound constraint type. For a primal upper bound constraint type (\leq), the corresponding negative dual variable can be motivated in a similar way. Regarding a strict equality constraint ($=$), the dual variable can have a positive or negative sign, which can be explained by the fact that the strict equality constraint can be replaced by two inequality constraint: one of lower and one of upper bound type. If one slightly decreases the lower bound (or increases the upper bound), the optimization routine has some room in the solution space to move to the upper or the lower bound, so that, depending on which of the two inequalities is binding, the sign of the dual variable is determined.

Table 1: Equivalences between primal and dual problem

Primal problem	Dual problem
Minimizing objective function	Maximizing
Objective function coefficient	RHS side coefficient of dual constraint
RHS side coefficient of primal constraint	Objective function coefficient
Coefficient matrix	Transposed coefficient matrix
Primal constraint	Dual variable
Constraint of type \geq	$y_i \geq 0$
Constraint of type \leq	$y_i \leq 0$
Constraint of type $=$	y_i no restriction on sign
Primal variable	Dual constraint
$x_j \geq 0$	Constraint of type \leq
$x_j \leq 0$	Constraint of type \geq
x_j no restriction on sign	Constraint of type $=$

2.1.2 Key characteristics of duality

The interactions between the primal and dual problem are described by two theorems the so-called weak and strong duality theorem, respectively, and the complementary slackness theorem. Since the connections between the primal and dual solution are important to understand and interpret the dual solution, the key results of the two theorems will be shortly outlined in the following.

Weak and strong duality theorems

The weak duality theorem states that the objective function value of the dual problem provides an upper bound for the primal objective function:

$$\sum_{j=1}^n c_j x_j \leq \sum_{i=1}^m b_i y_i \quad (3)$$

This theorem can be derived by a sequence of inequality relationships (Vanderbeigh 1997):

$$\sum_{j=1}^n c_j x_j \leq \sum_{j=1}^n \left(\sum_{i=1}^m y_i a_{ij} \right) x_j = \sum_{i=1}^m \left(\sum_{j=1}^n a_{ij} x_j \right) y_i \leq \sum_{i=1}^m b_i y_i \quad (4)$$

The strong duality theorem states that the optimal primal and the optimal dual objective function values are identical:

$$\sum_{j=1}^n c_j x_j = \sum_{i=1}^m b_i y_i \quad (5)$$

A proof of this theorem is given in Chvatál 1983.

Complementary slackness theorem

Introducing slack and surplus variables in the primal and dual problem formulation (1) and (2) results in the following presentation of the problems:

$$\text{Min} \sum_{j=1}^n c_j x_j \quad (6)$$

$$\sum_{j=1}^n a_{ij} x_j - w_i = b_i$$

$$x_j \geq 0$$

$$w_i \geq 0$$

with

$$w_i = \text{surplus variable } i.$$

$$\text{Max} \sum_{i=1}^m b_i y_i \tag{7}$$

$$\sum_{j=1}^n a_{ij} y_i + z_j = c_j$$

$$y_i \geq 0$$

$$z_j \geq 0$$

with

$$z_j = \text{slack variable } j.$$

Based on these formulations of the primal and dual problem, the complementary slackness theorem now states, that the optimal solution x and y of the primal and dual solution has to satisfy the conditions:

$$x_j z_j = 0 \tag{8}$$

$$w_i y_i = 0$$

For a proof of this theorem the reader is referred to Vanderbeigh 1997. The complementary theorem expresses that if the surplus variable of a constraint of the primal problem is unequal zero, i.e., the constraint is non-binding, the corresponding dual variable has to be zero. In other words, if the constraint would represent a limited resource, which is not yet fully exhausted, the dual problem would only assign a value of zero to this resource. On the other hand, if the resource is exhausted, i.e. the surplus variable becomes zero, the dual variable and thus the value of the resource can be greater than zero. The situation may occur that both the surplus and the dual variable are zero (so-called primal degenerated solution). This indicates that the constraint could be removed from the problem without changing the optimal solution of the problem, which implies that the constraint is redundant under the current problem settings (due to changes in some input parameters, e.g. fuel prices, this situation may change, so that the constraint should not be discarded or removed from the model in general).

2.2 Dual Solution of TIMES Model

The dual variable of a constraint quantifies its impact on the objective function and presents therefore valuable information for understanding and interpreting scenario results as well as debugging and improving the model itself. In the following interpretations for the dual variables of the main TIMES equations will be given. As a quick orientation, the constraints of the primal matrix and typical economic interpretations of their shadow prices are summarized in Table 2. A more detailed treatment for the interpretation of the individual dual variables is presented in the sub-sections. It should be noted that the interpretations of the dual variable given here assume that the optimization goal of the TIMES model is a minimization of the total discounted system costs. Since TIMES also allows the use of alternative user-defined objective functions, the interpretation may not necessarily be true for these alternative objective functions (as a prerequisite the alternative objective must still be measured in cost terms).

Table 2: Structure of TIMES primal constraint system

	VAR_ACT	VAR_CAP	VAR_COMNET	VAR_COMPRD	VAR_DEM	VAR_ELAST	VAR_FLO	VAR_IRE	VAR_NCAP	VAR_SIN	VAR_SOUT	Equation sign	RHS constant (excluding possible constants from fixed variables or past investments)	Description of Equation	Interpretation of Dual Variable
Objective function	+/- 4		+/-	+/-		+/-	+/-	+/-	+/-					Total discounted energy system costs over the model horizon	
EQ_ACTFLO	+						-					=	0	Definition of activity variable	Lowest price from all commodities defining activity
EQ(I)_ASHAR	+/-						+/-					\leq \geq	0	Annual share of group of flows in a larger group of flows on the input or output side of process	Certificate system between involved groups of process flows
EQ(I)_ACTBND	+											\leq \geq	ACT_BND	Bound on activity variable	Subsidy/tax on activity
EQE_ACTEFF	-						+/-					=	0	Activity-efficiency transformation equation	Cost impact of change of the inverse of ACT_EFF by 1/VAR_ACT
EQ(I)_BNDNET			+									\leq \geq	COM_NET	Bound on net production of commodity	Subsidy/tax on net production
EQ(I)_BNDPRD				+								\leq \geq	COM_PRD	Bound total production of commodity	Subsidy/tax on total production; Marginal production costs
EQ(I)_CAPACT	+								-			\leq	0	Capacity-activity utilization constraint	Capacity value; Part of capacity related costs covered in specific period and timeslice
EQ(I)_COMBAL	+/-		+/-	+/-	-	+/-	+/-	+/-	+/-	-	+	$=$ \geq	COM_PROJ*COMFR	Commodity balance	Commodity price; Marginal costs of increasing production or reducing consumption by one unit
EQE_COMPRD	+			-			+	+	+		+	=	0	Definition of total commodity production	Marginal production costs

⁴ +, - or +/- are used in the table to indicate the sign of the variable coefficient in the constraint. +/- means that either different instances of the same variable type appear with different signs in the constraint or that the coefficient can be positive or negative depending on the input data of the model (e.g. taxes or subsidies in the objective function).

EQ(I)_CPT		+							-			IV \leq II	CAP_BND	Capacity bound or definition equation for capacity variable	Subsidy/tax on total capacity
	VAR_ACT	VAR_CAP	VAR_COMNET	VAR_COMPRD	VAR_DEM	VAR_ELAST	VAR_FLO	VAR_IRE	VAR_NCAP	VAR_SIN	VAR_SOUT	Equation sign	RHS constant (excluding possible constants from fixed variables or past investments)	Description of Equation	Interpretation of Dual Variable
EQ(I)_CUMNET			+									IV \leq II	COM_CUMNET	Cumulative commodity bound on net production of a commodity	Subsidy/Tax escalating with discount rate
EQ(I)_CUMPRD				+								IV \leq II	COM_CUMPRD	Cumulative commodity bound on total production of a commodity	Subsidy/Tax escalating with discount rate; Hotelling rent for limited commodities
EQ(I)_FLOBND	+						+	+				IV \leq II	FLO_BND	Bound on process flows	Subsidy/tax on flow
EQ(I)_FLOFR	+/-						+/-					IV \leq II	0	Relationship between total annual flow and flow in a specific timeslice (load curves)	Cost impact on commodity price compared to case without load constraint
EQ(I)_FLOMRK	+			-			+/-	+	+		+	IV \leq II	0	Market share of process flow in total production of commodity	Certificate price of certificate system involving all technologies producing considered commodity
EQ_IRE	+/-							+/-				=	0	Inter-regional exchange equation	Value of imported/exported commodity
EQ(I)_IREBND	+							+				IV \leq II	IRE_BND	Bound on trade of a commodity between two regions	Tax/Subsidy on traded commodity
EQ(I)_XBND	+							+				IV \leq II	IRE_XBND	Bound on total imports or exports of a commodity in a region	Tax/Subsidy on total import/export of a commodity
EQ_PEAK	-						-	-	+			\leq	0	Peaking equation	Premium on commodity price for consumption during peak time; premium paid to producers for providing secure capacity

EQ_PTRANS	+/-						+/-					=	0	Transformation equation	Cost impact of increasing the FLO_FUNC(c1,c2) between commodities c1 and c2 by 1/VAR_FLO c1
	VAR_ACT	VAR_CAP	VAR_COMNET	VAR_COMPRD	VAR_DEM	VAR_ELAST	VAR_FLO	VAR_IRE	VAR_NCAP	VAR_SIN	VAR_SOUT	Equation sign	RHS constant (excluding possible constants from fixed variables or past investments)	Description of Equation	Interpretation of Dual Variable
EQ(I)_IN/OUTSHR	+/-						+/-					IV <= =	0	Share of flow in group of flows on the in- or output side of a process	Certificate system between involved process flows; e.g. for upper bound share part of commodity price of one flow is used to subsidy flow of other(s).
EQ(I)_STGIN/OUT										+	+	IV <= =	STGIN/OUT_BND	Bound on storage input or output flow	Tax/Subsidy on traded commodity
EQ_STGTSS/IPS	+/-									-	+	=	STG_CHRG	Storage equation	Value of stored commodity
EQ(I)_UCxxx	+/-	+/-	+/-	+/-			+/-	+/-	+/-			IV <= =	UC_RHSxxx	User-constraint	Due to flexible nature of constraint, interpretation depends on concrete UC (see text for 2 typical cases)
Direct variable bounds	ACT_BND	CAP_BND	COM_BNDNET	COM_BNDPRD			FLO_BND		NCAP_BND	STGIN_BND	STG_OUTBND				

2.2.1 Activity definition EQ_ACTFLO

Equation typ: Strict equality (=)
Dual variable: no sign restriction
Involved primal variables: VAR_FLO (-), VAR_ACT (+)

Interpretation:

The dual variable equals the absolute lowest price of the commodities (section 2.2.7) defining the activity variable.

2.2.2 Activity bound EQ(I)_ACTBND

Equation typ: Depending on specified activity bound (=, >=, <=)
Dual variable: no sign restriction for =, non-negative for >=, non-positive for <=

Involved primal variables: VAR_ACT (+)

Interpretation:

The dual variable describes in the case of a lower (upper) bound the cost increase (decrease) caused by an increase of the activity bound by one unit. For a fixed bound, the dual variable can be positive or negative. In this case, one can mentally replace the fixed bound constraint by lower bound and an upper bound constraint. If the dual variable of the fixed bound is positive, it represents a cost increase and it can be concluded that the lower bound constraint is binding. Vice-versa, if the dual variable of the fixed bound is negative, the upper bound constraint is binding.

The constraint is only generated in case, that the bound parameter is specified on a coarser timeslice level than the process timeslice level. Otherwise, the bound can be directly applied to the variable (section 2.2.23).

2.2.3 Activity-based efficiency relationship EQE_ACTEFF

Equation typ: Strict equality (=)
Dual variable: no sign restriction
Involved primal variables: VAR_FLO (+/-), VAR_ACT (-)

Interpretation:

The following transformation illustrates the impact of an increase of the RHS of the efficiency relationship for a simple process (one input commodity *cin* and one output commodity *cout*) by 1 unit:

$$VAR_FLO_{r,v,t,p,cin,s} - \frac{1}{ACT_EFF_{r,t,p,cin,in,s}} \times VAR_ACT_{r,v,t,p,s} = 1$$

$$VAR_FLO_{r,v,t,p,cin,s} - \frac{1}{ACT_EFF_{r,t,p,cin,in,s}} \times VAR_ACT_{r,v,t,p,s} - 1 = 0$$

$$VAR_FLO_{r,v,t,p,cin,s} - \left(\frac{1}{ACT_EFF_{r,t,p,cin,in,s}} + \frac{1}{VAR_ACT_{r,v,t,p,s}} \right) \times VAR_ACT_{r,v,t,p,s} = 0$$

Accordingly, the RHS increase and thus the dual variable can be regarded as the cost impact of an increase of the inverse of the efficiency ACT_EFF by the amount of $\frac{1}{VAR_ACT_{r,v,t,p,s}}$.

2.2.4 Activity bound EQ(l)_BNDNET

Equation typ: Depending on specified commodity net bound (=,>=,<=)

Dual variable: no sign restriction for =, non-negative for >=, non-positive for <=

Involved primal variables: VAR_COMNET (+)

Interpretation:

The constraint puts a bound on the net production (production minus consumption) of a commodity. The dual variable of this constraint can be interpreted for example for an upper bound as the total system cost reduction caused by an increase of the upper bound by one unit. An example for an upper bound constraint is the specification of bound on the net emission of CO2 into the atmosphere. Since CO2 is not only produced, but can be also stored, i.e. consumed, in sinks, the net production is relevant here. The dual variable then describes the marginal mitigation costs.

2.2.5 Activity bound EQ(l)_BNDPRD

Equation typ: Depending on specified commodity production bound (=,>=,<=)

Dual variable: no sign restriction for =, non-negative for >=, non-positive for <=

Involved primal variables: VAR_COMPRD (+)

Interpretation:

The constraint puts a bound on the production of a commodity. Hence, the dual variable of this constraint can be regarded as the marginal production costs of the commodity (see also section 2.2.8).

2.2.6 Capacity-Activity constraint EQ(I)_CAPACT

Equation typ: Lower bound constraint (\geq), Strict equality (=)

Dual variable: non-positive for \geq constraint, no sign restriction for strict equality constraint

Involved primal variables: VAR_ACT (+), VAR_NCAP (-)

Interpretation:

The dual value describes the cost reduction caused by an additional capacity unit and can thus be interpreted as the value of the capacity. For a power plant for example, it can be regarded as the part of the electricity price, which can be used for covering the fixed operating and investment costs of the capacity (as shown below in section 3.2). Since availability factors can be specified in addition to the process timeslice level on higher timeslice levels (e.g. for a DAYNITE process level on the SEASONAL or the ANNUAL level by NCAP_AFS or NCAP_AFA, respectively), also dual variables for these capacity-activity constraints on the coarser timeslice level may exist. The constraints on the coarser level offer a certain degree of flexibility, so that generation can be shifted from one process timeslice to another one as long as the overall constraint on the coarser level is satisfied (in addition an upper default availability factor of 1 is applied for the process timeslices, if no NCAP_AF factors are specified by the user). Therefore, the dual variable of these constraints on the higher timeslice levels can be regarded as the value or economic rent of a capacity unit including the flexibility of moving generation between timeslices. This means for example for a DAYNITE power plant that, in order to shift scarce capacity (annual capacity-activity constraint due to NCAP_AFA) from one process timeslice to another one, the electricity price in the target timeslice level must be high enough to cover the opportunity cost of the scarce capacity on the annual level.

2.2.7 Commodity balance EQ(I)_COMBAL

Equation typ: Lower bound constraint (\geq), Strict equality (=)

Dual variable: non-positive for \geq constraint, no sign restriction for strict equality constraint

Involved primal variables: VAR_ACT (+/-), VAR_FLO (+/-), VAR_IRE (+/-), VAR_COMNET (+/-), VAR_COMPRD (+/-), VAR_SIN (-), VAR_SOUT (+)

Interpretation:

The dual variable (shadow price) of the commodity balance describes the internal value of the commodity. It is therefore often referred to as the commodity price. If the commodity balance is binding, i.e., consumption equals production, the shadow price describes the cost change in the objective function induced by an increase of the commodity demand by one unit. Since the LHS of the commodity balance describes the difference between production and consumption, this additional demand may be covered by an increase in production or by a decrease in consumption. In the first case the shadow price is determined by activities on the supply side of the commodity, while in the latter case saving or substitution measures on the demand side of the commodity are setting the shadow price. Note, that when a peaking constraint (EQ_PEAK) for the considered commodity exists, the price consumers must pay during peak hours depends not only on the shadow price of the commodity balance but also on the shadow price of the peaking constraint (in case, that the

A commodity price of zero is often an indicator for an overproduction of the commodity due to some constraints or bounds on the production side leading to a production irrespectively of demand. Since such a model behavior is typically not desired, screening the model results for zero commodity prices helps to identify and correct these problematic areas in the model formulation. To keep the model feasible, i.e. avoiding contradicting constraints, one can introduce dummy imports for each commodity with very high import prices. If the model uses these dummy imports, it indicates a infeasible constraint set. The high import prices for the dummy imports then impact the commodity prices. Studying the model in the area close to these high commodity prices often helps to identify the cause for the infeasibility.

2.2.8 Commodity production definition EQE_COMPRD

Equation typ: Lower bound constraint (\geq), Strict equality ($=$)

Dual variable: non-positive for \geq constraint, no sign restriction for strict equality constraint

Involved primal variables: VAR_ACT (+), VAR_FLO (+), VAR_IRE (+), VAR_NCAP (+), VAR_SOUT (+), VAR_COMPRD (-)

Interpretation:

The dual variable describes the increase of the system costs, if the commodity production is increased by one unit. Hence, the dual variable represents the marginal production costs for one additional unit of the considered commodity. In comparison, the dual variable of the commodity balance is not restricted to the production, but includes also the consumption of the commodity. Hence, the dual variable of the commodity balance can represent the marginal cost for an additional production or for a demand reduction by one unit.

2.2.9 Capacity bound EQ(l)_CPT

Equation typ: Depending on specified capacity bound ($=, \geq, \leq$)

Dual variable: no sign restriction for $=$, non-negative for \geq , non-positive for \leq

Involved primal variables: VAR_NCAP (-), VAR_CAP(+)

Interpretation:

In case a capacity bound parameter CAP_BND has been specified for a process, the constraint EQ(l)_CPT puts a bound on the total installed capacity of said process (there are other cases, in which this equation is used to define the capacity variable VAR_CAP, if needed for other constraints). Then, the dual variable describes for a lower bound constraint the cost increase caused by an increase of the bound by one capacity unit. Similarly, for an upper bound constraint, the dual variable gives the cost decrease caused by a unit increase of the upper bound. For a fixed bound, the argumentation is similar as for a fixed activity bound constraint (EQ(l)_ACTBND, in section 2.2.2). A positive dual variable of the fixed bound constraint indicates that, if the fixed bound constraint would be replaced by an upper

2.2.10 Cumulative production/net consumption bound for a commodity EQ(I)_CUMPRD/NET

Equation typ: Depending on specified cumulated commodity bound
(=,>=,<=)

Dual variable: no sign restriction for =, non-negative for >=, non-positive
for <=

Involved primal variables: VAR_COMPRD (+), VAR_COMNET(+)

Interpretation:

The dual value of the constraint describes the change in the objective function, if the bound parameter is increased by one unit. The increase of an upper bound yields a reduction of the total costs (dual value is negative), since the usage of the commodity represents an attractive option. The increase of a lower bound yields an increase of the total costs (dual value is positive), since the system has to be forced to use more of an uncompetitive commodity (the commodity itself or the technologies utilizing it maybe too expensive). The dual value of a cumulative production constraint can also be interpreted as a tax/subsidy that is applied between the year y1 and y2 to reach the same cumulative production as specified in the bound. Therefore, the tax/subsidy has to be escalated by the discount rate starting from the year y1 to the last year y2.

If an upper cumulative commodity bound (COM_CUMPRD) is used to describe the available reserve/resource of a fossil primary energy carrier over the model horizon, the dual variable of the cumulative production constraint represents the value of the reserve. It can be interpreted as scarcity or resource rent and is called in depletion economic theory also Hotelling rent. Starting from the first year y1 (typically the beginning the model horizon), the Hotelling rent rises over time with the general discount rate. The Hotelling rent depends on various factors, a crucial factor one are the costs for substituting the scarce resource, once it gets depleted in the future.

2.2.11 Bound on process flow EQ(I)_FLOBND

Equation typ: Depending on specified flow bound (=,>=,<=)

Dual variable: no sign restriction for =, non-negative for >=, non-positive
for <=

Involved primal variables: VAR_FLO (+), VAR_ACT(+), VAR_IRE (+)

Interpretation:

The interpretation is very similar to the one for the EQ(I)_ACTBND constraint (section 2.2.2). The dual variable describes in the case of a lower (upper) bound the cost increase (decrease) caused by an increase of the flow bound by one unit. For a fixed bound, the dual variable can be positive or negative. In this case, one can mentally replace the fixed bound constraint by lower bound and an upper bound constraint. If the dual variable of the fixed bound is positive, it represents a cost increase and it can be concluded that the lower bound

The constraint is only generated in case, that the bound parameter is specified on a coarser timeslice level than the flow variable's timeslice level. Otherwise, the bound can be directly applied to the variable (section 2.2.23).

2.2.12 Bound on process flow EQ(1)_FLOFR

Equation typ: Depending on specified fractional load curve bound
(=,>=,<=)

Dual variable: no sign restriction for =, non-negative for >=, non-positive for <=

Involved primal variables: VAR_FLO (+/-), VAR_ACT(+/-)

Interpretation:

The constraint allows the specification of load curves for individual flows, specifying which share of the annual flow has to take place during a specific timeslice. The dual variable for a specific timeslice quantifies the additional costs contained in the commodity price for that timeslice, which are caused by the load specification in comparison to the situation without this load constraint. This interpretation can be illustrated by a small example shown in Figure 1 for fixed FLO_FR bound. The demand commodity DEM is balanced on an ANNUAL level, while the demand device DEVICE as well as the electricity input are depicted on a DAYNITE level. For simplicity, the demand device is assumed to have an efficiency of 100 %.

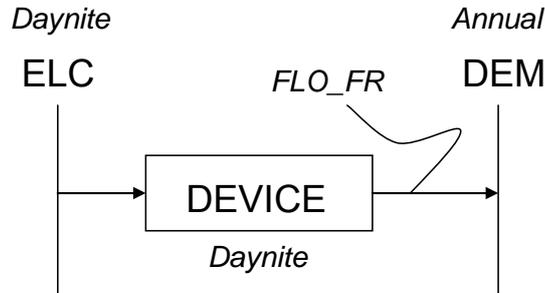


Figure 1: Example for demand device with FLO_FR load curve constraint

The corresponding dual constraint for the output flow variable in a timeslice s is:

$$EQG_COMBAL.M_{r,t,DEM,ANNUAL} - EQG_COMBAL.M_{r,t,ELC,s} - (1 - FLO_FR_{r,t,DEVICE,DEM,s,FX}) \times EQE_FLOFR.M_{r,t,DEVICE,DEM,s,FX} = 0$$

In the dual constraint, the commodity price of the electricity input commodity $EQG_COMBAL.M_{r,t,ELC,s}$ is split into two components: the price of the annual demand $EQG_COMBAL.M_{r,t,DEM,ANNUAL}$ commodity and the price component from the load curve constraint. The annual price of the demand commodity is a quantity-weighted average price over the entire year. The coefficient $(1-FLO_FR)$ reflects the fact, that the load curve influences already the annual commodity price of the demand, hence, not the entire shadow price of the load constraint is effective. The dual variable $EQE_FLOFR.M_{r,t,DEVICE,DEM,s,FX}$ describes the cost impact on the electricity price in timeslice s relative to the situation without this load constraint.

It should be noted that typically a load curve is specified for not just one timeslice, but typically for all. Therefore, the dual constraint above would consist of additional dual variables representing the impact on the commodity price in timeslice s from load curve specifications in timeslices other than s .

2.2.13 Bound on process flow EQ(I)_FLOMRK

Equation typ: Depending on specified market share bound (=,>=,<=)

Dual variable: no sign restriction for =, non-negative for >=, non-positive for <=

Involved primal variables: VAR_FLO (+/-), VAR_ACT(+), VAR_IRE (+), VAR_NCAP(+), VAR_SOUT(+), VAR_COMPRD (-)

Interpretation:

The constraint is a market share constraint for a commodity flow of a process. Its dual variable can be regarded as certificate price. For a lower bound constraint, the flow in the numerator of the market share receives a subsidy being $(1-FLO_SHAR)$ times the dual variable, while all other output flows producing the same commodity are penalized with costs of FLO_SHAR times the dual variable. For an upper bound, the compensation system is the other way round: the flow in the numerator is penalized, while all other output flows with the same commodity are subsidized.

2.2.14 Inter-regional exchange equation EQ_IRE

Equation typ: Strict equality (=)

Dual variable: no sign restriction

Involved primal variables: VAR_IRE (+/-), VAR_ACT(+/-)

Interpretation:

The shadow price of the inter-regional exchange equation quantifies for exchange processes the value of the imported and exported commodity. Often, this value equals in absolute terms the commodity price (dual variable of commodity balance, section 2.2.7) of the commodity in the import region, as can be verified by the dual constraint of the primal import flow variable:

$$VAR_IRE_{r,v,t,p,c,imp,s} \perp EQG_COMBALM_{r,t,c,s} + EQ_IRE.M_{r,t,p,c,imp,s} = 0$$

2.2.15 Bound on process flow EQ(I)_IREBND

Equation typ: Depending on specified inter-regional trade bound (=,>=,<=)

Dual variable: no sign restriction for =, non-negative for >=, non-positive for <=

Involved primal variables: VAR_IRE (+), VAR_ACT(+)

Interpretation:

The constraint allows the specification of bounds for the trade of a commodity between two regions. The dual variable describes in the case of a lower (upper) bound the cost increase (decrease) caused by an increase of the flow bound by one unit. For a fixed bound, the dual variable can be positive or negative. In this case, one can mentally replace the fixed bound constraint by lower bound and an upper bound constraint. If the dual variable of the fixed bound is positive, it represents a cost increase and it can be concluded that the lower bound constraint is binding. Vice-versa, if the dual variable of the fixed bound is negative, the upper bound constraint is binding.

2.2.16 Bound on process flow EQ(I)_XBND

Equation typ: Depending on specified inter-regional trade bound (=,>=,<=)

Dual variable: no sign restriction for =, non-negative for >=, non-positive for <=

Involved primal variables: VAR_IRE (+), VAR_ACT(+)

Interpretation:

The constraint allows the specification of bounds for the total import or export of commodity into or out of a region. The dual variable describes in the case of a lower (upper) bound the cost increase (decrease) caused by an increase of the flow bound by one unit. For a fixed bound, the dual variable can be positive or negative. In this case, one can mentally replace the fixed bound constraint by lower bound and an upper bound constraint. If the dual variable of the fixed bound is positive, it represents a cost increase and it can be concluded that the lower bound constraint is binding. Vice-versa, if the dual variable of the fixed bound is negative, the upper bound constraint is binding.

2.2.17 Peaking equation EQ_PEAK

Equation typ: lower bound constraint >=

Dual variable: non-negative

Involved primal variables: VAR_IRE (+/-), VAR_FLO (+/-), VAR_ACT(+/-), VAR_NCAP(+/-), typically VAR_NCAP has a positive sign and all other variables a negative one

Interpretation:

The dual value of the peaking equation describes the premium consumers have to pay in addition to the commodity price (dual variable of EQ(1)_COMBAL) during the peak timeslice. The premium equals $(1+COM_PKFLX)*FLO_PKCOI*RTCS_TSFR*(\text{dual variable})$. This premium is used to cover part of the investment costs of technologies providing secure capacity during the peak load timeslice (see also section 2.2.7).

2.2.18 Transformation equation EQ_PTRANS

Equation typ: Strict equality (=)

Dual variable: no sign restriction

Involved primal variables: VAR_ACT (+/-), VAR_FLO (+/-)

Interpretation:

Due to the flexibility of the transformation equation, the interpretation of its dual value depends on the specific case and does not always yield to a very illustrative interpretation. In general, the transformation equation links the input and the output side of a process. Therefore, its dual variable passes information on the cost terms from the input to the output side and vice-versa. For a simple case, a process with one input flow c1 and one output flow c2 being linked by an efficiency FLO_FUNC(c1,c2), the dual variable, which is being defined as the cost change when the RHS is increased by one unit, can be interpreted as cost change when the efficiency of the process is increased by the amount of $1/VAR_FLO(r,v,t,p,c1,s)$:

$$\begin{aligned}
 & VAR_FLO_{r,v,t,p,c2,s} - FLO_FUNC_{r,v,t,p,c1,c2,s} \times VAR_FLO_{r,v,t,p,c1,s} = 1 \tag{9} \\
 & VAR_FLO_{r,v,t,p,c2,s} - FLO_FUNC_{r,v,t,p,c1,c2,s} \times VAR_FLO_{r,v,t,p,c1,s} - 1 = 0 \\
 & VAR_FLO_{r,v,t,p,c2,s} - \left(FLO_FUNC_{r,v,t,p,c1,c2,s} + \frac{1}{VAR_FLO_{r,v,t,p,c1,s}} \right) \times VAR_FLO_{r,v,t,p,c1,s} = 0
 \end{aligned}$$

2.2.19 Input/output share constraint EQ(I)_IN/OUTSHR

Equation typ: Depending on specified share bound (=, >=, <=)

Dual variable: no sign restriction for =, non-negative for >=, non-positive for <=

Involved primal variables: VAR_ACT (+/-), VAR_FLO (+/-)

Interpretation:

The dual value describes for a lower bound, the subsidy needed to guarantee that the flow is at the given lower bound. The subsidy is needed, since for an output flow the shadow price of the produced commodity is too low to cover the production costs of the flow variable (for an input flow the opposite is true, the commodity is too expensive to be used in the process). The value of the subsidy, which the flow receives, is equal to $(1 - \text{FLO_SHAR}) * (\text{dual variable})$. This subsidy has to be paid by the other flows forming the denominator in FLO_SHAR constraint, thus, the costs for these flows are increased by the amount $\text{FLO_SHAR} * (\text{dual variable})$. In a similar way, an upper bound FLO_SHAR can be interpreted as a tax being added to the costs of a flow.

2.2.20 Bound on storage flow EQ(l)_STGIN/OUT

Equation type: Depending on specified share bound (=, >=, <=)

Dual variable: no sign restriction for =, non-negative for >=, non-positive for <=

Involved primal variables: VAR_SIN (+), VAR_SOUT (+)

Interpretation:

The constraint allows one to put bounds on the input or output flows of a storage process. The interpretation is therefore very similar to the one of the EQ(l)_FLOBND constraint (section 2.2.11)

2.2.21 Storage equation EQ_STGTSS/IPS

Equation type: strict equality (=)

Dual variable: no sign restriction

Involved primal variables: VAR_SIN (-), VAR_SOUT (+), VAR_ACT(+)

Interpretation:

For a storage process, the dual variable of the storage equation (EQ_STGTSS for a storage process operating between timeslices, EQ_STGIPS for a storage process operating between periods) represents the unit value of the commodity being stored. Due to factors as costs and losses associated with the storage, this value may be different to the commodity price (dual variable of the commodity balance EQ(l)_COMBAL for the stored commodity).

2.2.22 User constraints

Equation typ: Depending on specified share bound (=, >=, <=)

Dual variable: no sign restriction

Involved primal variables: VAR_ACT (+/-), VAR_FLO (+/-), VAR_COMNET (+/-), VAR_COMPRD (+/-), VAR_NCAP (+/-), VAR_IRE(+/-)

Interpretation:

User constraints in TIMES are very flexible constraint type, which can be used to formulate virtually any kind of constraint. Therefore, no general valid interpretation of the dual variable of a constraint can be given. Instead two types of frequently occurring user constraints are discussed here: an absolute bound type of user constraint and a relative share type of user constraint.

Absolute bound type of user constraint

This type of user constraint puts an absolute bound on a sum of variables (VAR_ACT , VAR_FLO , VAR_NCAP , VAR_CAP , VAR_COMNET , VAR_COMPRD), which may be mixed and may have different coefficients. The constraint can be specified explicitly for certain periods, timeslices and regions, or it can be cumulative over these dimensions. The RHS bound of the user constraint is specified by one of the $UC_RHS(R)(T)(S)$ parameters. The RHS parameter choice depends on whether the constraint should be generated for specific periods, timeslices or regions or whether it is cumulative over one of these dimensions.

The dual variable of an absolute type of user constraints represents the cost change in the primal objective function when the RHS bound is increased by one unit. Depending on the constraint type, this cost change can be positive or negative. For a binding lower bound constraint (\geq) the objective function will increase, whereas for a binding upper bound constraint (\leq) an increase in the RHS parameter results in a cost decrease. For a fixed bound type of constraint, the dual variable can be positive or negative. If it is positive, it implies that the fixed bound constraint could be replaced by a lower bound type of user constraint, while a negative value indicates that the strict equality constraint can be replaced by an upper bound type of constraint.

In case that the user constraint is cumulative over time, the interpretation can be similar to the interpretation of the cumulative production bound for a commodity (see section 2.2.10).

Share type of user constraint

An example for a share type of user constraint is the formulation of renewable quota q in electricity generation:

$$\frac{\sum_{prc \in REN} \sum_s VAR_FLO_{r,v,t,prc,ELC,s}}{\sum_{prc \in REN} \sum_s VAR_FLO_{r,v,t,prc,ELC,s} + \sum_{prc \in NONREN} \sum_s VAR_FLO_{r,v,t,prc,ELC,s}} \geq q_{r,t} \quad (10)$$

$$(1 - q_{r,t}) \sum_{prc \in REN} \sum_s VAR_FLO_{r,v,t,prc,ELC,s} - q_{r,t} \sum_{prc \in NONREN} \sum_s VAR_FLO_{r,v,t,prc,ELC,s} \geq 0$$

with

REN	=	set of renewable electricity generation technologies,
$NONREN$	=	set of non-renewable electricity generation technologies,
ELC	=	electricity commodity.

The dual variable of the quota can be interpreted as the price for renewable electricity certificates and represents the difference between the electricity price (dual variable of the electricity commodity balance) and the generation costs of the most expensive renewable technology still needed to fulfill the quota. The renewable generator receives an amount of

$(1-q)$ *x certificate price* for the renewable electricity in addition to the market price, whereas the non-renewable electricity supplier has to buy q certificates for each unit of non-renewable electricity produced.

2.2.23 Direct bounds on variables

Bound parameters: ACT_BND, CAP_BND, COM_BNDNET/PRD, FLO_BND, NCAP_BND, STGIN/OUT_BND

Interpretation:

If the bound parameters are specified by the user on the timeslice level, at which the variable is balanced in TIMES, e.g. the activity variable on the process timeslice level, the bounds are directly applied to the variable and constraints, as the EQ(I)_ACTBND constraint for the activity, are not needed⁵.

If a primal decision variable is at its lower bound, the corresponding dual variable can be regarded as the necessary subsidy, so that the decision variable leaves its lower bound. In case of an upper bound, the dual variable can be regarded as the opportunity costs earned by the difference between the value of the decision variable and its costs, e.g. for an upper bound on the new capacity variable for a power technology the difference between the revenue from electricity sales per capacity unit and its investment costs. The dual variable of the upper bound can be also regarded as the necessary tax level so that the primal does not exceed the upper bound. The dual variables associated with direct bounds on primal variables are often called *reduced costs*. Background for this name is the interpretation that the dual variable quantifies also the amount by which the cost coefficient of the primal variable has to be reduced in the objective function, so that the primal variable leaves its lower bound.

Since all primal variables are non-negative, even if no lower bound has been explicitly specified, each variable has a lower bound of zero. If a primal variable is zero, one gets a reduced costs information for this primal decision, illustrating how far away the decision is from becoming cost-competitive.

For new capacity bounds (NCAP_BND), the original reduced costs are corrected to account for the fact that the salvage value of capacity installation exceeding the end of the model horizon is subtracted in the objective function from the total capacity-related costs of the new capacity addition. This salvage value term in the objective function is a factor leading to decreasing reduced costs when the periods are approaching the end of the model horizon. To get a meaningful interpretation of the reduced costs for these cases, a correction factor has been introduced in the TIMES result processing for VEDA-BE, so that the reduced costs for the new investment variable can be interpreted as the necessary cost reduction of the specific investment costs parameter (NCAP_COST).

⁵ If the bound parameters are specified on a finer timeslice level than the variable, the bound parameters are first aggregated to the variable level by an internal preprocessing routine in TIMES.

3. PRICE FORMATION EQUATIONS

In the previous chapter, interpretations for the individual dual variables of a TIMES model have been given. In addition, valuable information can also be obtained from analyzing the interactions between the different dual variables, which are given by the constraints of the dual problem. In this chapter, therefore, first some general aspects of the dual constraints of a TIMES model are discussed before then two special dual equations are discussed in more detail. Finally, the usage of the information obtained from the dual equations to perform a cost-benefit analysis is presented.

3.1 Price Formation Equations (dual constraints)

The dual problem shown in standard form with slack variables in (11) establishes a consistent constraint system, by which the dual variables are determined. Each variable of the primal problem corresponds with one equation of the dual problem. The complementary slackness theorem (8) states that, if the primal variable is greater zero, the slack variables $z_{up,j}$ and $z_{lo,j}$ of the dual constraint j have to be zero. This means that the terms involving the dual variables on the LHS of (11) equal the cost coefficient of the primal variable. This can be interpreted in such a way that the net value derived from the primal variable, which equals the LHS term of the dual constraint, has to cover the costs associated with the primal variable.

$$Max \sum_{i=1}^m b_i y_i + \sum_{j=1}^n up_j z_{up,j} + \sum_{j=1}^n lo_j z_{lo,j} \quad (11)$$

$$\sum_{j=1}^n a_{ij} y_i + z_{up,j} + z_{lo,j} = c_j$$

$$y_i \geq 0$$

$$z_{up,j} \geq 0$$

$$z_{lo,j} \leq 0$$

with

$$up_j = \text{upper bound on primal variable } x_j,$$

$$lo_j = \text{lower bound on primal variable } x_j,$$

$$z_{up,j} = \text{slack variable linked to the upper bound on the primal variable } x_j,$$

$$z_{lo,j} = \text{slack variable linked to the lower bound on the primal variable } x_j.$$

The slack variables $z_{up,j}$ and $z_{lo,j}$ represent the reduced costs mentioned in section 2.1.1. If the primal variable is at its lower bound (which is in most cases zero, if the user has not explicitly specified another lower bound for the variable), the slack variable $z_{lo,j}$ is typically

c_j $z_{up,j}$

The LHS term $\sum_{i=1}^n a_{ij} y_i$ of the dual constraint in eqn. (11) can be further split in a term representing all positive contributions $(a_{ij} y_i)^+$ and a term consisting of all negative terms $(a_{ij} y_i)^-$:

$$\text{Max} \sum_{i=1}^m b_i y_i + \sum_{j=1}^n up_j z_{up,j} + \sum_{j=1}^n lo_j z_{lo,j} \quad (12)$$

$$\sum_{j=1}^n (a_{ij} y_i)^+ + \sum_{j=1}^n (a_{ij} y_i)^- + z_{up,j} + z_{lo,j} = c_j$$

$$y_i \geq 0$$

$$z_{up,j} \leq 0$$

$$z_{lo,j} \geq 0$$

The negative terms can be regarded as costs caused by the primal variable, whereas the positive values can be interpreted as revenues derived from the primal variable. For example a coal power plant consumes coal and produces electricity. The dual constraint of the activity variable then has a negative term representing the cost of the coal consumption being equal the coal price (dual variable of coal commodity balance) times the coal usage per activity unit and a positive one representing the revenue from the electricity produced being equal the electricity price times the electricity generation per activity unit. If the primal variable is a cost-effective decision, the revenue term must be equal or exceed the cost term plus the cost coefficient as stipulated in eqn. (13):

(13)

$$\sum_{j=1}^n (a_{ij} y_i)^+ \geq c_j - \sum_{j=1}^n (a_{ij} y_i)^-$$

If the revenues exceed the total costs, the primal variable is at its upper bound and the difference between the two terms is compensated by the upper slack variable in eqn (12).

By analyzing the dual equations for the optimal solution, one can identify the influence of the dual variables (and thus the primal constraints) and of the primal cost coefficients on the

⁶ Even if the primal decision variable is zero, situations may occur where the slack variable is also zero. This corresponds to a so-called primal degenerate solution as discussed in section 2.1.2.

costs and revenues associated with a primal decision variable. Thus, one can deduce why a certain primal variable is part of the optimal solution or not. Among the various dual constraints, two types of dual constraints are especially important: one is the dual constraint associated with the activity variable VAR_ACT and the other one is the constraint linked to the new capacity VAR_NCAP. The former constraint helps to explain why a certain technology is or is not cost-effective under the prevailing scenario conditions, while the latter illustrates how the investment costs for building new capacity for a technology are recovered over the lifetime of the technology.

3.2 Price Formation Equation of the Activity Variable

The dual equation of the activity variable allows for a large number of technologies the analysis of its cost-competitiveness. The main idea behind this analysis is to compare the value created by the technology's activity with its related costs. The value of the activity is typically represented by the value of the output commodities produced. For a coal power plant, for example, the dual variable or price of the produced electricity represents the value of the activity, while the price of the coal input commodity would be part of the costs associated with the activity (Figure 2).

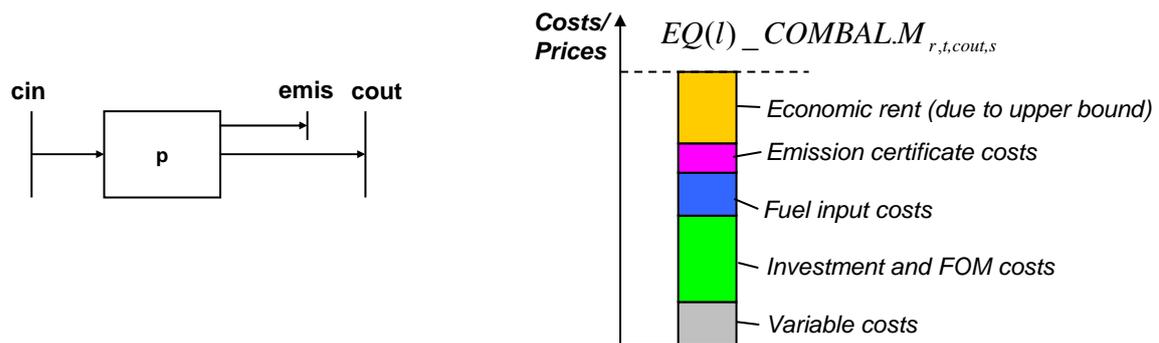


Figure 2: Typical components of the dual constraint for the activity variable of a technology

An emission, although an output commodity, can create costs, if an emission bound or a tax has been specified for it. Other cost components typically occurring are the variable operating costs (excluding the fuel input costs) and the dual variable of the capacity-activity constraint, representing the part of the investment and fixed operating and maintenance costs which have to be paid in the current time period.

An example of the dual constraint for an activity variable of the process shown in Figure 2 is given in eqn. (14). The first three terms on the LHS represent costs associated with the input commodity, the fixed and investment costs, the emissions. The fourth term represents the revenue for producing the output commodity. For simplicity it has been assumed here that the process and the commodities are modeled on the same timeslice level, if this is not the case additional coefficients reflecting the different timeslice resolution appear in (14). If the activity variable is at its lower or upper bound, corresponding terms representing the reduced costs (lower bound) or the economic rent (upper bound) are present. In addition, the activity variable might be involved in further constraints, e.g. user constraints, so that the dual variables of these constraints would appear as additional cost or value terms in the dual equation. It should be noted that the sign (+/-) of the different terms in (14) stems from

the sign of the coefficient of the activity variable in the different primal constraints. Depending on the constraint type ($\leq, \geq, =$), the dual variable can be positive or negative, so that the overall product of coefficient and dual variable determines the sign of the term.

$$\begin{aligned}
 & - \sum_{cin} \frac{1}{ACT_EFF_{r,t,p,cin,s}} EQG_COMBAL.M_{r,t,cin,s} \\
 & + EQL_CAPACT.M_{r,v,t,p,s} \\
 & + \sum_{emis,cin} \left(FLO_EMIS_{r,t,p,emis,cin,s} \frac{1}{ACT_EFF_{r,t,p,cin,s}} EQE_COMPRD.M_{r,t,emis,s} \right) \\
 & + EQG_COMBAL.M_{r,t,cout,s} \\
 & + VAR_ACT.M_{r,v,t,p,s,UP} \\
 & + VAR_ACT.M_{r,v,t,p,s,LO} \\
 & \dots \\
 & = \\
 & act_cst_{r,t,p,s} \\
 & \perp VAR_ACT_{r,v,t,p,s}
 \end{aligned}
 \tag{14}$$

}

Cost terms

Values

Reduced costs/Economic rent

Cost coefficient of activity variable

with

$act_cst_{r,t,p,s}$

cost coefficient of activity variable in primal objective function (note: cost coefficient is discounted and represents the costs for the entire period duration and thus is different from ACT_COST),

$ACT_EFF_{r,t,p,cin,s}$

activity-based efficiency of converting the input flow commodity cin into the flow(s) defining the activity, emission factor quantifying how much emissions of $emis$ are produced per unit of input cin ,

$EQ_COMBAL.M_{r,t,cin,s}$

dual variable of the commodity balance of input cin ,

$EQ_COMBAL.M_{r,t,cout,s}$

dual variable of the commodity balance of input $cout$,

$EQL_CAPACT.M_{r,v,t,p,s}$

dual variable of the capacity-activity constraint of process p in period t and timeslice s for capacity being built in vintage period v ,

$EQE_COMPRD.M_{r,t,emis,s}$

dual variable of the fixed commodity bound production constraint for the emission $emis$ in period t (the constraint

$VAR_ACT.M_{r,v,t,p,s,UP}$ defines in intermediate variable for the emission produced VAR_COMPRD , the emission bound parameter COM_BNDPRD is then applied directly as bound to the variable VAR_COMPRD),
 $VAR_ACT.M_{r,v,t,p,s,LO}$ reduced costs (or better economic rent) associated with the upper bound on the activity variable,
 $VAR_ACT_{r,v,t,p,s}$ reduced costs associated with the lower bound on the activity variable,
activity variable,

For example, the commodity balance EQG_COMBAL is a lower bound type of constraint, so that according to Table 1 the dual variable is positive. Hence, the dual term in eqn. (14) associated with the input commodity is a cost term due to its overall negative sign, whereas the dual term of the output commodity is positive and represents a value associated with the activity variable.

Eqn. (14) can be rearranged in such a way that the value terms appear on one side and all the cost terms on the other side:

$$\begin{aligned}
& EQG_COMBAL.M_{r,t,cout,s} && (15) \\
& + VAR_ACT.M_{r,v,t,p,s,UP} \\
& + VAR_ACT.M_{r,v,t,p,s,LO} \\
& \dots \\
& = \\
& act_cst_{r,t,p,s} \\
& + \sum_{cin} \frac{1}{ACT_EFF_{r,t,p,cin,s}} EQG_COMBAL.M_{r,t,cin,s} \\
& - EQL_CAPACT.M_{r,v,t,p,s} \\
& - \sum_{emis,cin} \left(FLO_EMIS_{r,t,p,emis,cin,s} \frac{1}{ACT_EFF_{r,t,p,cin,s}} EQE_COMPRD.M_{r,t,emis,s} \right) \\
& \dots \\
& \perp VAR_ACT_{r,v,t,p,s}
\end{aligned}$$

This equation can be interpreted in such a way that, if the activity variable not at its upper or lower bound the value of the output commodity (LHS term) must cover its associated

costs (RHS term). If the activity variable is at its lower bound, e.g. at its lower bound of zero, the value of the output commodity bound does not cover the costs, hence, the activity is zero. To fulfill eqn. (15) with equality the missing cost coverage is compensated by the reduced cost term associated with the lower bound ($VAR_ACT.M_{r,v,t,p,s,LO}$). On the contrary, if the value terms exceed the costs, the activity variable represents a very attractive decision in terms of minimizing the total system costs. However, an upper bound on the activity variable prevents the optimization routine to set the activity variable to a higher value. The cost advantage of the activity variable is captured by the economic rent associated with the upper bound ($VAR_ACT.M_{r,v,t,p,s,UP}$), so that eqn. (15) is fulfilled with equality. Thus, the analysis of the dual variable for the activity variable allows to quantify whether the usage of a technology represents a cost-effective option or, if not, how far the technology is away from becoming cost-effective.

Prerequisite for using the dual constraint of the activity variable in such a way is, however, that the internal reduction algorithm is activated, which replaces for the example process shown in Figure 3 the input and output flow variables by the activity variable, so that the dual variables of the input and output commodity appear in one constraint. If the reduction algorithm is not used, the several dual constraints have to be used to analyze the technology. Similarly, in case of a flexible technology, i.e. a technology where the mix of several input or output flows is determined by the optimization process, the analysis of several dual constraints associated with the different flow variables associated with the process is required.

If the reduction algorithm is not activated in TIMES, the process shown in Figure 3 will have in addition to the activity variable a flow variable for the input and one for the output flow. The resulting set of dual equations for these three variables is:

$$VAR_ACT_{r,v,t,p,s} \perp \quad (16)$$

$$EQ_ACTFLO.M_{r,v,t,p,s} + EQL_CAPACT.M_{r,v,t,p,s} \\ - \frac{I}{ACT_EFF_{r,t,p,cin,s}} \cdot EQE_ACTEFF.M_{r,v,t,p,cin,in,s} \\ = \\ 0$$

$$VAR_FLO_{r,v,t,p,cin,s} \perp$$

$$FLO_EMIS_{r,t,p,emis,cin,s} \cdot EQE_COMPRD.M_{r,t,emis,s} + EQE_ACTEFF.M_{r,v,t,p,cin,in,s} \\ - EQG_COMBAL.M_{r,t,cin,s} \\ = \\ 0$$

$$VAR_FLO_{r,v,t,p,cout,s} \perp$$

$$- EQ_ACTFLO.M_{r,v,t,p,s} + EQG_COMBAL.M_{r,t,cout,s} = 0$$

By solving the first equation for $EQ_ACTFLO.M$ and the second for $EQE_ACTEFF.M$ and substituting into the third one, the resulting constraint is identical to the dual constraint (14) of the activity variable. This example illustrates that for some more complex technologies, not a single constraint, but a set of several constraints has to be considered to analyze the value and cost terms related to a technology in detail. For an aggregate analysis of the costs and benefits as shown below in section 3.4, it would have been sufficient here to analyze from the constraint set in (16) only the last constraint, since it contains the dual variable of the commodity balance of the main output commodity (cout), which for many technologies represents its associated main benefit. This benefit is then mirrored with the costs, summarized, here, by the dual variables $EQ_ACTFLO.M$ and $EQE_ACTEFF.M$. The impact of individual constraints, e.g. the emission bound, is not apparent, but hidden in the $EQE_ACTEFF.M$ dual variable. To give at least the possibility to analyze dual constraints, though not an automatized fashion, more interactively, a browser for the matrix and the primal and dual solution terms is presented in section 4.

3.3 Price Formation Equation of the New Capacity Variable

The dual equation associated with the new capacity variable is given in eqn (17). The RHS of the constraint consists of the total costs associated with the new capacity variable divided into the total discounted fixed operating and maintenance costs over the lifetime of the technology as well as the total discounted investment costs.

(17)

$$- \sum_{t \geq v} \sum_{s \in \text{prc_ts}}^{\text{v+TLIFE}} \alpha_{r,v,t,p,s} \cdot \text{EQL_CAPACT.M}_{r,v,t,p,s}$$

$$+ \sum_{t \geq v} \sum_{c \in \text{output of p}} \sum_{s \in \text{com_ts}}^{\text{v+TLIFE}} \beta_{r,v,t,p,s} \cdot \text{EQ_PEAK.M}_{r,t,c,s}$$

$$+ \text{VAR_NCAP.M}_{r,v,p,UP}$$

$$+ \text{VAR_NCAP.M}_{r,v,p,LO}$$

...

=

$$\text{inv_cst}_{r,v,p} + \text{fom_cst}_{r,v,p}$$

$$\perp \text{VAR_NCAP}_{r,v,p}$$

with

$$\alpha_{r,v,t,p,s} = \text{rtp_cptyr}_{r,v,t,p} \cdot \text{prc_capact}_{r,p} \cdot \text{af}_{r,v,t,p,s} \cdot \text{g_yrfr}_{r,s},$$

$$\beta_{r,v,t,p,s} = \frac{\text{rtp_cptyr}_{r,v,t,p} \cdot \text{prc_capact}_{r,p} \cdot \text{af}_{r,v,t,p,s} \cdot \text{g_yrfr}_{r,s}}{1 + \text{com_pkrsv}_{r,t,c}},$$

$$\text{inv_cst}_{r,t,p,s}$$

investment cost coefficient of new capacity variable in primal objective function (note: cost coefficient is discounted and represents the costs over the entire lifetime),

$$\text{fom_cst}_{r,t,p,s}$$

fixed operating and maintenance cost coefficient of new capacity variable in primal objective function (note: cost coefficient is discounted and represents the costs over the entire lifetime),

These costs for new capacity have to be covered by the value terms on the LHS of the equation. The first term containing the dual variable EQ(I)_CAPACT.M represents the value associated with the usage of the technology. For example, for a power plant this term can be regarded as cost coverage stemming from some portion of the electricity price. The second term containing the dual variable of the peaking equation EQ_PEAK.M quantifies the part of the cost coverage from the fact that the capacity provides secure capacity during peak load. In addition, if lower or upper bounds are present, the corresponding reduced costs or economic rents indicate in case of a binding lower bound the part of the costs not covered by the value terms and in case of a binding upper bound the amount, by which the value terms exceed the RHS costs, i.e. the capacity investment is very attractive.

The LHS sums the value terms over the entire lifetime of the new installed capacity. Analyzing the value terms for the periods and thus their distribution over time gives an indication, in which periods larger part of the costs can be recovered.

3.4 Cost-Benefit Analysis

The purpose of the cost-benefit analysis is a standardized analysis of the costs and benefits associated with a primal variable. As discussed in section 3.1, the terms of the dual constraints can be split in entries representing costs and entries representing values or benefits linked to the primal decision variable. The comparison of the costs and the benefits allows an assessment of the cost-competitiveness of a primal decision variable. From eqn. (12) the costs are summarized as the sum all negative dual terms plus the cost coefficient of the primal variable in the primal problem. The benefits are represented by the sum of all positive dual terms:

$$costs = c_j - \sum_{j=1}^n (a_{ij} y_i)^- \quad (18)$$

$$benefits = \sum_{j=1}^n (a_{ij} y_i)^+$$

In the dual equations (12), a possible difference between the costs and the benefits is adjusted by negative reduced costs variables $z_{up,j}$ for upper bounds and positive variables $z_{lo,j}$ for lower bounds.

As described in (Finnis et al. 1984) for MARKAL the ratio of benefits and costs is used to evaluate a primal variable:

$$benefit - cost \text{ ratio} = \frac{\sum_{j=1}^n (a_{ij} y_i)^+}{c_j - \sum_{j=1}^n (a_{ij} y_i)^-} \quad (19)$$

A value greater of 1 indicates that the benefits exceed the costs, i.e. the primal decision represents a very attractive option. The primal variable would have been even larger, if its upper bound had been absent. If the benefit-cost ratio is smaller 1, the primal decision is not cost-competitive yet. In this case the value of the ratio shows how far the decision is away from becoming a cost-effective solution. Thus, the benefit-cost ratio is a helpful tool in the result analysis of energy system models, since a value close to, but still below, 1, indicates a technology that under the current scenario assumptions is not yet cost-competitive, but may become so due to some (smaller) changes in the input assumptions as investment costs for the considered technology or scenario assumptions as mitigation targets. In a pure analysis of the primal solution results, this technology would have went unnoticed, since its primal decision variables are zero and not part of the basic solution. Taking into account the uncertainty attached to many of the input assumptions in longer term scenarios, the use of the benefit-cost ratio may help to identify additional potential technologies deserving more attention, e.g. in the form of a sensitivity analysis.

Based on the experience with the benefit-cost ratio in MARKAL, the ratio calculation has been implemented within this undertaking as part of the matrix analysis tools for TIMES (see following chapter).

Using the benefit-cost analysis with TIMES models, revealed one anomaly for technologies being not cost-competitive, i.e. their activity and new capacity variables are zero (or at their lower bound). For these technologies, the correct benefit-cost ration appears with the new capacity variable (VAR_NCAP), whereas for the activity variable the ratio is always 1, although the activity is zero and not cost-effective. This behavior can be understood, when looking at the dual equations of the activity and new capacity variable (just the most basic dual variables from the commodity and capacity-activity constraints have been considered here for simplicity):

$$\begin{aligned}
& EQG_COMBAL.M_{r,t,cout,s} + VAR_ACT.M_{r,v,t,p,s,UP} + VAR_ACT.M_{r,v,t,p,s,LO} & (20) \\
= & \\
& act_cst_{r,t,p,s} + \sum_{cin} \frac{1}{ACT_EFF_{r,t,p,cin,s}} EQG_COMBAL.M_{r,t,cin,s} - EQL_CAPACT.M_{r,v,t,p,s} \\
& \perp VAR_ACT_{r,v,t,p,s} \\
& - \sum_{t \geq v} \sum_{s \in pre_ts} \alpha_{r,v,t,p,s} \cdot EQL_CAPACT.M_{r,v,t,p,s} - VAR_NCAP.M_{r,v,p,UP} + VAR_NCAP.M_{r,v,p,LO} & (21) \\
= & \\
& inv_cst_{r,v,p} + fom_cst_{r,v,p} \\
& \perp VAR_NCAP_{r,v,p}
\end{aligned}$$

In the dual constraint of the activity variable (20), the reduced cost term of the lower bound ($VAR_ACT.M_{r,v,t,p,s,LO}$) is zero, although the activity variable is at its lower bound of zero.

The commodity prices of the input and output commodities cin and $cout$ are determined by other activities in the model and can therefore be regarded as given. If the commodity price of the output commodity exceeds the variable operating costs plus the costs for the input commodity, the remaining portion of the output commodity price is assigned to the dual variable of the capacity-activity constraint, so that overall the benefits equal the costs and the benefit-cost ration is 1. The amount assigned to the dual variables of the capacity-activity constraints ($EQL_CAPACT.M_{r,v,t,p,s}$) is not enough to cover the capacity-related costs, which becomes evident in the dual constraint of the new capacity variable (21). Here, the reduced cost term $VAR_NCAP.M_{r,v,p,LO}$ is unequal zero and balances the too low benefits from the capacity-activity constraints with the cost terms. Hence, the benefit-cost ratio is smaller 1.

From this observation for technologies being not cost-competitive it can be concluded that only the benefit-cost ratio for the new investment variable provides reliable information. In principal, also the opposite case could be possible, i.e., the reduced cost term of the

investment variable $VAR_NCAP.M_{r,v,p,LO}$ is set to zero in (21) and then the dual variables of the capacity-activity constraints are set in such a way that they cover the entire capital costs. These dual variables are then put into (20), where then the costs exceed the benefits so that the reduced cost term for the activity variable ($VAR_ACT.M_{r,v,t,p,s,LO}$) is greater zero. In this case, the benefit-cost ratio of the activity variable is smaller 1 and the one for the new capacity variable is 1. Hence, only the benefit-cost ratio of the activity would be meaningful.

The author so far observed only the first case for non-competitive technologies, i.e., the benefit-cost ratio of the activity variables being at their lower bound is 1 (meaningless) and the one for the new capacity variable is smaller 1 (meaningfull).

4. MATRIX TOOLS

The goal of the matrix tools, which have been developed within this undertaking, is to provide the modeler with access to the primal and dual equations in a more user-friendly fashion as currently possible in the GAMS listing file⁷. In addition, the tools combine the primal and dual solution with the matrix coefficients, so that the individual cost and benefit terms of the dual constraint can be analyzed.

4.1 Overview of the matrix tools

Figure 3 shows the steps and tools involved in generating input files, so that starting from a TIMES model its matrix can be generated and selected (or all) primal and dual constraints can be analyzed in VEDA-BE.

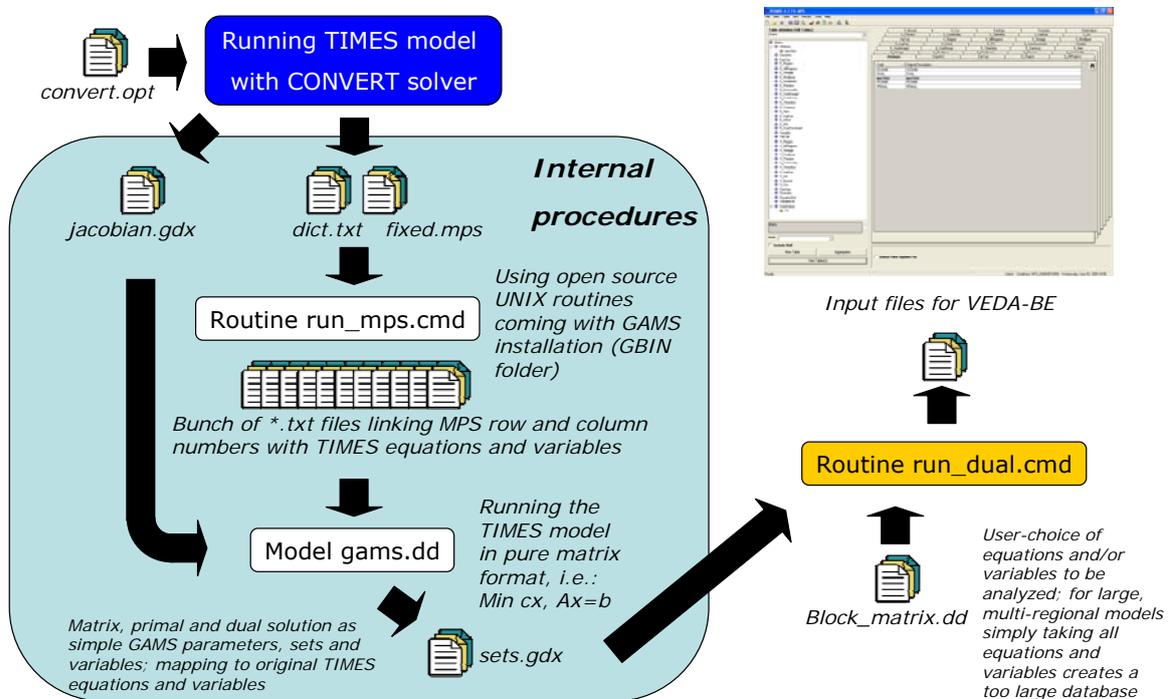


Figure 3: Overview of the tools to create the matrix files for VEDA-BE

Starting point of the routine is an existing TIMES model and its RUN file. In the RUN file the solver CONVERT, instead of one of the typical LP solvers (e.g. CPLEX, XPRESS), has to be chosen by modifying/adding the line to

```
OPTION LP=CONVERT;
```

When running now the TIMES model, it is not optimized, but three files are generated:

⁷ The GAMS lst file is generated within each model run and provides a list of all model constraints (row listing) and of all matrix columns in a text file (column listing).

- `jacobian.gdx`: it contains the first-order derivatives of the constraints with respect to the primal variables (Jacobi matrix). For linear constraints, these derivatives equal the matrix coefficients, so that basically, this `gdx` file contains the primal matrix of the problem. In addition, the `gdx` file contains information on the upper and lower bounds of the primal variables. The constraints and variables are numbered serially starting with `e1` and `x1`, respectively, so that the reference to the constraint and variable names is initially lost, and has to be reestablished using the dictionary file `dict.txt`.
- `dict.txt`: the dictionary file links the constraint and variable names used in the `jacobian.gdx` and the `fixed.mps` file with the names used in TIMES.
- `fixed.mps`: the file contains the optimization problem in the so-called MPS (mathematical programming system) format. The information on the RHS constants and the constraint type (`<=`,`=`,`>=`) is extracted from this file.

In a batch routine called `run_mps.cmd`, which takes the files `dict.txt` and `fixed.mps` as input, a set of text files in GAMS format is created linking constraint and variables numbers with TIMES names. These text files and the `jacobian.gdx` file are then used in a GAMS programme (`gams.dd`), which creates a GAMS problem in matrix format being equivalent to the original TIMES problem and then solves this problem. The resulting primal and dual solutions as well as the problem matrix are saved in the file `sets.gdx`. This information is then used in a third routine `run_dual.cmd` to transform user-selected parts of the primal and dual constraint matrix in combination with the related primal and dual solution into result attributes, which can be imported into VEDA-BE. The selection of the constraints and variables is done by the user through the text file `block_matrix.dd`. The file should contain sets called `BLK_(equation name)` and `BLK_(variable name)` indicating the rows and columns of the problem matrix, for which input attributes for VEDA-BE should be generated by the routine `run_dual.cmd`. For example the following excerpt of the `block_matrix.dd` file ensures that for the matrix part covered by the `EQE_ACTEFF` equations in the region `WEU` attributes for VEDA-BE are created:

```
LOOP(S_EQE_ACTEFF(eqn, 'WEU', vintage, modlyear, prc, comgrp, io, ts),
      BLK_EQE_ACTEFF('WEU', vintage, modlyear, prc, comgrp, io, ts) = YES;
);
```

The resulting input files for VEDA-BE are called `matrix.vd`, `matrix.vde` and `matrix.vds`.

From the software side, the utilities used in the first batch routine `run_mps.cmd` are coming all with the GAMS software installation (directory `GBIN` or `GBIN_NEW` in the GAMS folder). They are originally unix utilities, which have been ported to other platforms. Documentation for them can be found in the internet. The other two routines are, as mentioned, based on GAMS. So, no other software than GAMS and VEDA-BE is needed.

4.2 Attributes in VEDA-BE

Due to additional dimensions used for the VEDA-BE attributes (compared to MARKAL or TIMES VEDA-BE result attributes), one has to create first based on an existing set of `matrix.vd/vde/vds` files a new database. The database, once created, can then be used for the matrices of different models.

The matrix-related attributes available in VEDA-BE are MATRIX, PCOMBI, PRIMAL, DUAL and DCOMBI.

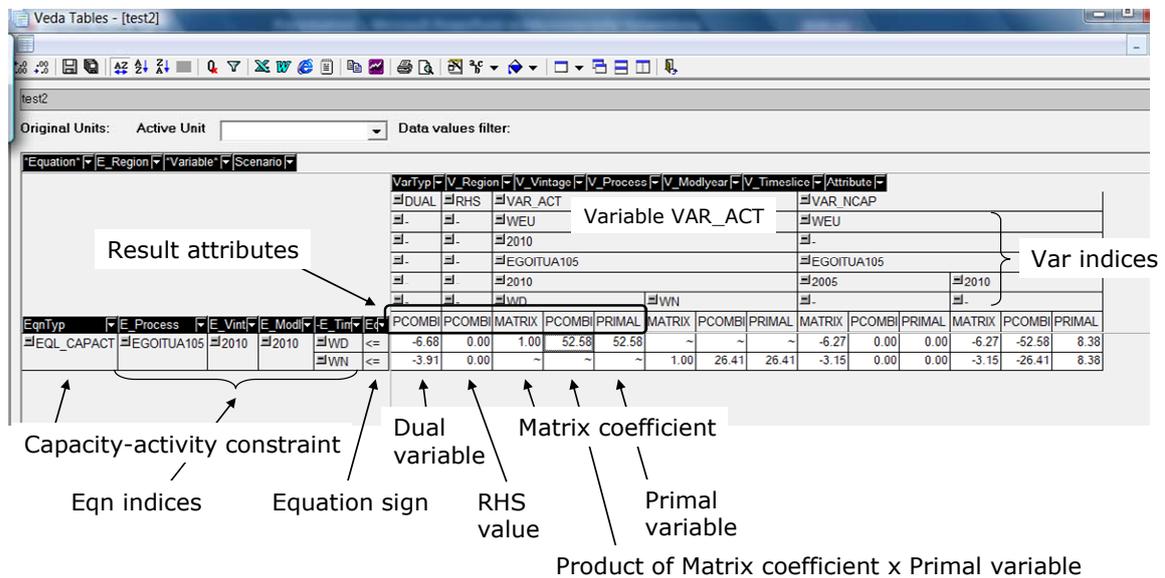
The MATRIX attribute contains the problem matrix of the TIMES model, i.e. the coefficients a_{ij} . Depending on the ordering of row and column indices in VEDA-BE, one can create with this attribute tables displaying the primal or dual constraints (Figure 4).

EqnTyp	VAR_ACT	VAR_CAP	VAR_COMN	VAR_COMP	VAR_FLO	VAR_JRE	VAR_NCAP
EGE_ACTEFF	-4,152	~	~	~	3,880	~	~
EGE_CAPACT	1,016	~	~	~	~	~	-645
EGE_COMBAL	496,327	~	-106	54	192,823	54	~
EGE_COMPRD	748	~	~	-54	795	8	~
EGE_CPT	~	52	~	~	~	~	-49
EGE_INSHR	~	~	~	~	-1,128	~	~
EGE_OUTSHR	0	~	~	~	-35	~	~
EQG_COMBAL	12,263	~	~	~	-2,246	900	~
EQG_CPT	~	~	~	~	~	~	-237
EQG_INSHR	~	~	~	~	19	~	~
EQG_OUTSHR	~	~	~	~	383	~	~
EQG_UCRSUS	790	~	~	~	~	~	~
EQG_UCRT	33	~	~	~	-363	~	~
EQG_UCRTS	288	~	~	-1	78	~	~
EQL_ACTBND	63	~	~	~	~	~	~
EQL_CAPACT	5,440	~	~	~	~	~	-10,148
EQL_CPT	~	~	~	~	~	~	-5
EQL_INSHR	~	~	~	~	47	~	~
EQL_OUTSHR	~	~	~	~	-19	~	~
EQL_UCR	1,008	~	~	~	~	~	~
EQL_UCRSU	269	6	~	~	~	~	~
EQL_UCRSUS	16	~	~	~	~	~	~
EQL_UCRT	349	28	~	~	252	~	~
EQL_UCRTS	-115	~	~	~	-457	~	~
EQL_XBND	~	~	~	~	~	3	~
EQ_ACTFLO	1,291	~	~	~	-1,452	-1,116	~
EQ_JRE	0	~	~	~	~	~	~
EQ_PEAK	-6	~	~	~	421	2	2,211
EQ_PTRANS	~	~	~	~	-38	~	~

Figure 4: Table displaying the primal matrix of TIMES model in aggregated form using the MATRIX attribute

To identify individual TIMES equations or variables similar dimensions are used in VEDA-BE as in GAMS. The only difference made for VEDA-BE is that the dimensions used for variables start with V_ and the ones for equations with E_, e.g. the dimension E_Process contains all processes of the model. In addition, the dimension VarTyp of the MATRIX attribute consists of all TIMES variables and the dimension EqnTyp lists all equations occurring in the model.

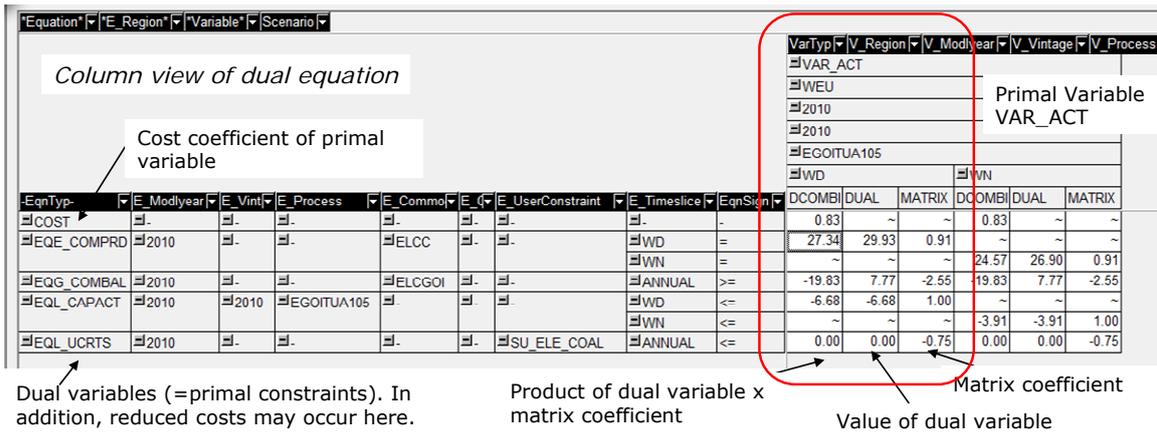
The PRIMAL attribute contains the optimal values for all primal variables, the DUAL attribute the values for the dual variables. The PCOMBI attribute contains for each matrix entry the product of matrix coefficient and primal variable. In addition, the VarTyp dimension of the PCOMBI attribute contains for each constraint its RHS and its dual variable value (VarTyp=RHS and VarTyp=DUAL). In Figure 5, the three attributes MATRIX, PCOMBI and PRIMAL have been combined in a table to give detailed information to individual constraints. To allow the primal values be displayed for each matrix entry, the PRIMAL attribute is from its structure a matrix with the primal solution values assigned to their occurrence in the constraint matrix.



$$EQL_CAPACT_{WEU,2010,2010,EGOITUA105,WD} : \\ VAR_ACT_{WEU,2010,2010,EGOITUA105,WD} - 6.27 \cdot VAR_NCAP_{WEU,2005,EGOITUA105} - 6.27 \cdot VAR_NCAP_{WEU,2010,EGOITUA105} \leq 0$$

Figure 5: Example of primal constraint in VEDA-BE (detailed view) using the MATRIX, PRIMAL, PCOMBI attributes

The dual constraint of a primal activity variable is shown in Figure 6. To generate the table in VEDA-BE, the attribute DCOMBI has been used, which contains the product of matrix coefficient and dual variable for each matrix entry, i.e. the value term of the corresponding primal variable with respect to the constraint of the dual variable.



<u>Dual equation for VAR_ACT</u>	<u>Dual variable</u>	<u>Value flows</u>
$0.91 \cdot M_{EQE_COMPRD}_{WEU,2010,ELLC,Wd}$	+ 29.93	$\Sigma = 0.83$ <ul style="list-style-type: none"> + 27.34 Electricity price revenue - 19.83 Fuel costs - 6.68 Contribution to inv and fixom costs - 0
$- 2.55 \cdot M_{EQG_COMBAL}_{WEU,2010,ELCGOI,Wd}$	+ 7.77	
$+ 1.00 \cdot M_{EQL_CAPACT}_{WEU,2010,2010,EGOITUA105}$	- 6.68	
$- 0.75 \cdot M_{EQL_UCRTS}_{WEU,2010,SU_ELE_COAL}$	0	
\leq		\leq
Cost coefficient of VAR_ACT		0.83 Variable operating costs

Figure 6: Example of dual constraint in VEDA-BE (detailed view) using the MATRIX, DUAL, DCOMBI attributes

For accessing the benefit cost-ratio of a variable, the attribute BENCOST can be used in VEDA. It contains under the dimension EqnTyp different entries: COST for the cost coefficient of the primal variable, NEG for the sum of all costs (cost coefficient plus negative value flows), POS for all positive value flows, RATIO for the benefit-cost ratio and, finally, PRIMAL for its primal value as well as DUAL for its reduced costs. As discussed in section 3.4, only the benefit-cost ratio of the new capacity variable seems to provide meaningful information.

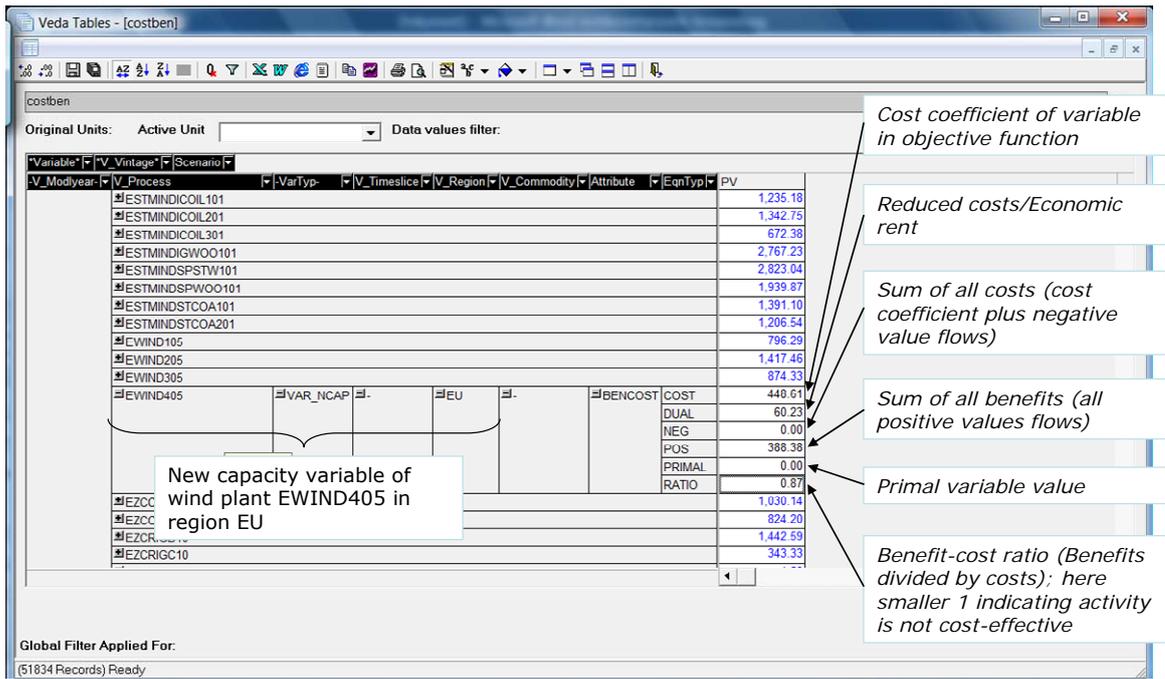


Figure 7: Example of benefit-cost ratios using the attribute BENCOST

5. SUMMARY AND OUTLOOK

The dual solution of a TIMES model provides valuable information to better understand the scenario results and thus provides deeper insight into a model and its results. Within this undertaking, first an economic interpretation of the different dual variables available from a TIMES model has been given. Parallel to the original problem formulation of a TIMES model (so-called primal problem) exists due to its LP nature, a second, so-called dual problem formulation, which describes the interactions of the different dual variables. These dual constraints allow an assessment of the cost-effectiveness of the primal decision variables and, thus, help to understand why certain decisions have been taken under the optimization regime. Two important dual constraints related to the activity and the new capacity variable have been discussed in more detail. The latter one can be used to define a benefit-cost ratio as indicator for cost-effectiveness of a technology. To give an easier and more interactive access to the primal and dual constraints of a TIMES model, tools have been developed allowing the transformation of the TIMES matrix in a format which can be imported into VEDA-BE. Within VEDA-BE, the entire model matrix can be displayed and from there selected sections of the primal and dual constraints as well as the benefits and costs associated with decision variables be analyzed in more detail.

The tools have been tested with larger models, as the TIAM model. While still operating also with these large-scale models, performance becomes an issue with increasing model size. The step of creating the VEDA-BE import files has been identified as main time-consuming bottle-neck. Currently, due to limits in the number of dimensions allowed by the GDX2VEDA tool the import files have been created by put routine. Future work may be needed to streamline this step. From the user perspective it would also be useful to include the information of the benefits and costs associated with a technology in the view of the reference energy system (RES). Thus, the modeler can compare in vertical direction competing technologies and can analyze in horizontal direction the evolution of commodity prices influenced by costs and benefits linked to technologies along a technology chain.

6. REFERENCES

- Chvatál, V.: Linear Programming, W.H. Freeman and Company, New-York, 1983
- Dantzig, G.B. and Thapa, M.N.: Linear Programming I: Introduction, Springer-Verlag, 1997
- Finnis, M.W., Gundermann, J., Tosato, G.: Value flows in a linear economic model, 1984
- Loulou, R., Remme, U., Kanudia, A., Lehtilä, A. and Goldstein, G.: Documentation for the TIMES Model, Energy Technology Systems Analysis Programme (ETSAP), April 2005, (<http://www.etsap.org/documentation.asp>)
- Vanderbeigh, R.J.: Linear Programming: Foundations and Extensions, Kluwer Academic Publishers, Boston, 1997