ENERGY TECHNOLOGY SYSTEMS ANALYSIS PROGRAMME

TIMES Grid Modelling Features

Final Report to ETSAP with details on the design of the features

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Foreword

This Report documents the features of the new TIMES extension for gas and electricity grid infrastructures.

The document is divided into three chapters. Chapter 1 contains an introduction with a brief literature review on electricity and gas grid modelling in large-scale energy systems models. Chapter 2 presents the philosophy and mathematical formulation for improving the representation of electricity grids via DC Power Flow Modelling with Endogenous Transmission Expansion. Chapter 3 presents the philosophy and mathematical formulation for improving the representation of gas grids via a suitably and computationally efficient linearised Weymouth equation.

This report complements the *User Note* for the ETSAP project, *TIMES Extension for electricity, gas, hydrogen, and CO2 transport infrastructures*. Evangelos Panos (PSI) designed the extension, with contributions from Blanche Brognart (PSI). Antti Lehtilä (VTT) further refined, validated, and implemented it into TIMES.

The TIMES user is directed to the *User Note* for the details on how to enable, parametrise and utilise the extension in TIMES instances. Instead, the current documentation is directed towards TIMES users who would like to understand the new features more deeply.

This documentation and the accompanying *User Note* supplement the comprehensive documentation of the TIMES model generator.

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1. INTRODUCTION

As countries strive towards net-zero targets, energy system models must increasingly account for the interdependence between electricity and gas networks. The growing penetration of variable renewable energy sources (VRES), electrification of demand, and the decarbonisation of gas supply chains all contribute to a tighter coupling between power and gas or hydrogen systems. Excluding the physical characteristics of these grids from long-term models risks underestimating constraints such as transmission bottlenecks, intertemporal flexibility, or infrastructure investment needs.

Incorporating grid constraints into long-term planning models, such as TIMES, is critical for assessing the adequacy, security, and cost-effectiveness of energy transitions. For instance, gas grids (natural gas, biomethane, biogas or hydrogen) can provide short-term flexibility through linepack storage. At the same time, electricity networks must accommodate geographically dispersed renewable energy sources and new demands, such as electric vehicles and heat pumps. Nonetheless, modelling these networks within TIMES has proven challenging due to its reliance on linear programming (LP) formulations, which ensure computational tractability across multi-decade horizons.

Both electricity and gas grid equations are inherently nonlinear. While power flow equations can be approximated using linear DC models—such as the phase-angle or power transfer distribution factor (PTDF) formulations—these are linear only under a fixed network topology. If endogenous grid expansion is to be considered—i.e., the model is allowed to select new transmission lines—the problem becomes non-linear, as the PTDF or susceptance matrices must be recalculated each time the grid structure changes (Gröwe-Kuska et al., 2003; Ruiz and Conejo, 2009). This challenge is typically addressed through approximations involving virtual lines and binary variables, resulting in a mixed-integer linear program (MILP) that only indirectly simulates grid expansion. Similarly, the steady-state behaviour of gas flow is governed by the Weymouth equation, which is quadratic and non-convex. Direct implementation of such equations would breach the LP structure of TIMES, necessitating the use of linear approximations or relaxations (Shin et al., 2022).

The TIMES community has partially addressed the representation of electricity grid networks of fixed topology via linear DC power flow equations. Based on (Lehtilä & Giannakidis, 2015), a spatially disaggregated analysis of transmission congestion and generation dispatch is represented. Outside of TIMES, other integrated energy models have explored more advanced mathematical formulations. For example, convex relaxations using semidefinite programming (SDP) or second-order cone programming (SOCP) have been employed for gas networks (He et al., 2018; Chen et al., 2019), while piecewise linear approximations are common in mixed-integer linear models (Ordoudis et al., 2019; Sirvent et al., 2017). Many of these models co-optimise electricity, gas and hydrogen operations, with gas/hydrogen-fired power plants and power-to-gas units acting as interface technologies.

By embedding grid constraints in a modular way, similar to the approach followed by (Lehtilä & Giannakidis, 2015), the TIMES extensions described in this report allow detailed network modelling without disrupting the existing structure of national energy systems. It allows electricity and gas infrastructure to be modelled as modular add-ons, preserving backwards compatibility with existing Reference Energy System (RES) structures while offering a detailed spatial and temporal representation of grid constraints.

This represents a significant advancement, enabling the TIMES framework to support more robust policy assessments and infrastructure planning in the face of sector coupling and increasing spatial complexity.

The specific grid modelling features of TIMES are tailored for modellers who wish to integrate basic characteristics of energy transmission grids into their energy system models. Modelling electrical, natural gas, biomethane, biogas, hydrogen, and CO_2 grids in TIMES requires that at least a simple representation of the geographical and spatial distribution of generators, consumer loads, and the network be included in the reference energy system. This needs to be done elegantly to reduce complexity.

The grid modelling features can be useful, for instance, in analysing potential bottlenecks in the electricity transmission system and evaluating the impacts of integrating large amounts of variable renewable generation into the system. Such analyses could, therefore, be utilised to indicate the enforcement and new investments needed in various parts of the main grid under different future scenarios. They can also be beneficial for representing gas, hydrogen, or CO_2 emissions pipeline networks by accounting for the physical properties of the gas flows within those pipelines.

It should be noted that whenever we refer to gas grids in the following discussion, we imply grids of any type of gaseous fuel, including natural gas, hydrogen, biogas, or biomethane, as well as emission grids, such as CO_2 emissions transport networks.

The specific grid modelling features include the following components:

- Linear DC power flow equations based on a phase-angle or PTDF formulation.
- Linear gas grid equations based on the Weymouth formulation.
- A facility for including a grid representation in the model by using an add-on approach with automated allocation of generation and demand to grid nodes.
- Simplified N–1 security constraints.
- New parameter for defining costs on specific grid flows.

The PTDF formulation for DC power flow equations and the Weymouth formulation for gas flow equations both support an MIP formulation option, which is needed for consistent modelling of transmission expansion planning and bidirectional pipelines in gas grids.

2. Modelling of Electricity Grids

This section presents the modelling of the transmission grid network using Power Transfer Distribution Factors (PTDF). Please note that TIMES already supports modelling the transmission grid network using phase-angle formulation. However, using the PTDF formulation allows for modelling transmission expansion planning by accounting for the induced changes in the grid operation. This requires calculating the PTDF factors for the entire system, including all lines, which implies a mixed-integer and non-linear problem.

2.1 Why transmission expansion requires PTDF recalculation

In the transmission expansion planning (TEP) context, accurately representing how power injections affect line flows is essential for ensuring physically meaningful and economically efficient investment decisions. The Power Transfer Distribution Factors (PTDFs) are central to this representation, as they define the linear sensitivity of line flows concerning nodal injections and withdrawals. However, these coefficients are topology-dependent: any change in the transmission network—such as the addition, removal, or reinforcement of transmission lines—modifies the electrical characteristics of the grid and necessitates a recalculation of the PTDF matrix.

PTDF coefficients are derived from the inverse of the nodal susceptance matrix, which is itself a function of a) the location and connectivity of transmission lines, b) the electrical impedance of those lines (typically their reactance), and c) the overall structure of the network. Any change in the network, such as adding a new line or changing the capacity of an existing one, alters the susceptance matrix and, therefore, requires recalculating the PTDFs. Failing to do so results in flow approximations that no longer reflect the physical behaviour of the updated grid. Using outdated PTDF values when the grid topology has changed leads to a) incorrect estimation of power flows across the network, b) misleading identification of congestion or underutilisation, c) distortion of locational marginal costs, and d) potentially infeasible real-world operational conditions. This misrepresentation directly undermines the reliability of the TEP solution, as flow constraints and investment trade-offs are based on flawed sensitivities. Transmission expansion is inherently aimed at alleviating congestion and enhancing network resilience. Because each new line alters the distribution of flows across the grid, recalculated PTDFs are essential to a) identify new congestion patterns post-expansion, b) determine the marginal value of additional transmission capacity, and c) enable cost-optimal dispatch of generation units across space and time.

Within TIMES, a full recalculation of PTDFs during optimisation would imply a nonlinear and nonconvex problem formulation. To preserve tractability, the PTDF-based TEP design in TIMES adopts a precomputed PTDF matrix that includes both existing and candidate lines.

The status of each candidate line is then managed through:

- virtual flow variables (to represent the contribution of inactive lines), and
- binary investment variables (to control line activation using MIP logic – a linear approximation relaxation is also possible and provided).

This approach enables consistent approximation of network effects within a linear or mixed-integer linear program, balancing physical accuracy with computational efficiency. The focus is exclusively on transmission lines, omitting other transmission elements. The lines are modelled with DC power flow and do not account for power losses. The extension can also be used for modelling the electricity grid at different voltage levels.

2.2 Design of the PTDF-based Transmission Expansion Planning

We adopt the PTDF-based transmission expansion planning (TEP) formulation proposed by Rahmani et al. (2016), in which both existing and candidate transmission lines are represented within a precomputed PTDF matrix. This approach avoids recalculating the PTDF matrix for every line investment decision. In addition, it eliminates the need for voltage angle variables by linearly relating nodal power injections to line flows concerning a fixed slack bus, preserving the physics of DC power flow while reducing the number of variables and constraints. To ensure that lines not yet built do not carry physical flow, the method introduces virtual flows and bounding constraints that suppress flow contributions from uninstalled lines.

In this approach, a virtual line is a mathematical artefact that captures the non-physical flow which would arise on a candidate line if the PTDF were applied without constraints. These virtual flows are not real; they represent the erroneous flows that the PTDF formulation would assign to unbuilt lines due to their inclusion in the matrix.

Binary decision variables (or their linear relaxation) indicate whether a candidate line is installed. If a line is not built, its associated virtual flow—decoupled from the physical grid—is allowed. When a line is selected for investment, the virtual flow is deactivated and replaced by a real flow consistent with the PTDF formulation. This disjunctive structure allows candidate line contributions to be handled in a physically meaningful and computationally efficient way.

In this regard, for each candidate line, a virtual flow variable is introduced. The actual flow on the line is then split into two components:

- A real flow variable, which becomes active if and only if the line is installed.
- A virtual flow variable, which exists to cancel or suppress the artificial flow from the PTDF when the line is not installed.

In other words, and as will be detailed in the next section, the TIMES model imposes a constraint on these virtual flows in the form of:

$$\left|p_{i,j,t}^{virt}\right| \leq \left(1 - y_{i,j,t}\right) \cdot M \cdot \overline{P}_{i,j}^{max} \qquad \textit{eq. 1}$$

where $p_{i,j,t}^{virt}$ is the virtual flow of the line connecting nodes i and j, $y_{i,j,t} \in \{0,1\}$ is a binary variable representing whether the line is built, $\overline{\mathrm{P}}_{\mathrm{i,j}}^{\mathrm{max}}$is the maximum allowed flow capacity, and M is a so-called big M constant. When the line is not built, that is when $y_{i,j,t} = 0$, the constraint is meant to be inactive, through the choice of a sufficiently large value for M, allowing any level for the virtual flow, while the zero line capacity is forcing the real flow to be zero. Once the line is installed, $y_{i,j,t} = 1$ and the constraint forces the virtual flow to become zero, allowing the real flow to be defined according to the active PTDF matrix while it is also constrained by the capacity installed.

This mechanism allows the model to use a fixed PTDF matrix throughout the optimisation, maintain linear constraints and avoid recalculating PTDFs, to accurately enforce investment decisions online availability and avoid infeasible or physically meaningless flows.

Compared to angle-based models, the PTDF-based formulation significantly reduces model size and improves scalability, particularly when assessing large candidate lines. The resulting TEP problem is compact and tractable, supporting a linear or mixed-integer linear (MILP) structure. While rooted in the existing TIMES grid formulation, this extension replaces the angle-based power flow equation with a PTDF representation to better facilitate large-scale planning.

2.2.1 Mathematical formulation

The general form of the DC load flow equations can be written as follows:

$$g_i - d_i = \sum_j p_{ij} = a_i \qquad \text{eq. 2}$$

Where g_i is the power injected into the node i by generators and d_i is the power withdrawn from the node i by consumer loads while $p_{i,j}$ is the branch flow from node i to node j. It follows that a_i is the net injection at the node i.

When using the voltage angle formulation, it holds that:

$$p_{i,j} = (\delta_i - \delta_j)/X_{i,j} \qquad \text{eq. 3}$$

Where $X_{i,j}$ is the reactance of the branch connecting nodes i and j, and δ_i is the voltage phase angle of the node i with respect to a reference angle. The above constraint was already implemented in the previous extension of TIMES, which represents electricity grids. Using the PTDF formulation, the branch flow from node i to node j can be calculated as:

$$p_{i,j,t,s} = \sum_{k \in N} PTDF_{i,j,k} \cdot a_{k,t,s} \qquad \text{eq. 4}$$

Where $PTDF_{i,j,k}$ is the Power Transfer Distribution Flow matrix, and $a_{k,t,s}$ is the net injection at the node k in the year t and timeslice s.

We now assume that the PTDF matrix includes all possible candidate transmission lines, i.e., we compute the PTDF matrix as if all these lines are already installed. Then, we eliminate any transmission lines that are not physically present yet by using virtual injections.

Let's introduce a new variable $p_{i,j,t,s}^{virt}$ that is a virtual flow on a candidate line ij in timeslice s. According to Rahmani et al. (2016), we can write valid constraints for the real and virtual flows as follows:

$$\left|p_{i,j,t,s} - \sum_{k \in N} PTDF_{i,j,k} \cdot a_{k,t,s} - \sum_{(m \to n) \in \mathcal{L}} \left(PTDF_{i,j,m} - PTDF_{i,j,n}\right) \cdot p_{m,n,t,s}^{virt}\right| \leq M \times (1 - y_{i,j,t,s}) \quad \forall ij \in \Omega \qquad \text{eq. 5}$$

$$\left|p_{i,j,t,s}^{virt} - \sum_{k \in N} PTDF_{i,j,k} \cdot a_{k,t,s} - \sum_{(m \to n) \in \mathcal{L}} \left(PTDF_{i,j,m} - PTDF_{i,j,n}\right) \cdot p_{m,n,t,s}^{virt}\right| \leq M \times y_{i,j,t,s} \quad \forall ij \in \Omega \qquad \text{eq. 6}$$

$$\left|p_{i,j,t,s}\right| \leq M \times y_{i,j,t,s}$$
$$\left|p_{i,j,t,s}^{virt}\right| \leq M \times (1 - y_{i,j,t,s}) \qquad \text{eq. 7}$$

As one can see, the constraints are analogous for the real flows and the virtual flows, and the equation (5) is active only when $y_{i,j,t,s} = 1$, while equation (6) is active only when $y_{i,j,t,s} = 0$. Based on the above discussion and the methodology shown by Rahmani et al. (2016), we can now combine equations (5) and (6) and thereby rewrite equation (eq. 4) to account for the virtual flows from candidate new lines in the grid topology.

$$p_{i,j,t,s} + p_{i,j,t,s}^{virt} = \sum_{k \in N} PTDF_{i,j,k} \cdot a_{k,t,s} + \sum_{(m \to n) \in \mathcal{L}} \left(PTDF_{i,j,m} - PTDF_{i,j,n}\right) \cdot p_{m,n,t,s}^{virt} \qquad \text{eq. 8}$$

In the above equation, the notation $m \to n$ denotes a line direction and $\mathcal{L}$ is the set of lines. The above equation avoids recomputing the PTDF matrix when the grid's topology changes because of an investment decision. Instead, all candidate lines connecting new nodes m and n are included upfront, even if they are not yet installed. The virtual flow variables absorb the flow component associated with these candidate lines that are not yet present. These flows are zeroed out if the line is built and are bounded if the line is unbuilt (to avoid unintended physical flows). This approach ensures that PTDF consistency is preserved with investment decisions that change grid topology without requiring non-linear updates to the matrix.

Instead of the actual injections $a_{k,t,s}$, we can define virtual node injections as follows:

$$a_{k,t,s}^{VRT} = a_{k,t,s} + \sum_{(k \to n) \in \mathcal{L}} p_{k,n,t,s}^{virt} - \sum_{(m \to k) \in \mathcal{L}} p_{m,k,t,s}^{virt} \qquad \text{eq. 9}$$

Since in the TIMES formulation, the nodal balances with the actual net injections are already imposed by the EQ_COMBAL equations, and therefore, taking into account equation (9), we can further simplify the equation (eq. 8) into the following:

$$p_{i,j,t,s} + p_{i,j,t,s}^{virt} = \sum_{k \in N} PTDF_{i,j,k} \cdot a_{k,t,s}^{VRT} \qquad \text{eq. 10}$$

As the PTDF equations of form (4) are known to satisfy the nodal balances (apart from a total balance), with the introduction of equation (10) those balances are satisfied when including the virtual flows. However, from (eq. 9) one can see that the actual balances will then likewise be satisfied. The equation (10) therefore defines the DC power flow on transmission lines using the PTDF formulation, while accounting for the possibility that the line is a candidate line that has not yet been built. The term $\sum_{k\in N} PTDF_{i,j,k} \cdot a_{k,t,s}^{VRT}$ represents the theoretical flow on the line (i,j) in the system and reflects the linear influence of each node's injection on every line. If the line (i,j) is a candidate (i.e., not yet built), then any flow implied by $a_{k,t,s}^{VRT}$ via the PTDF must be cancelled unless the line is installed. The cancellation is done by introducing the $p_{i,j,t,s}^{virt}$, a "dummy" variable that subtracts away the flow from the real network physics if the line does not yet exist. Only if the line is installed will $p_{i,j,t,s}^{virt}$ be forced to zero in equation (eq. 10). Therefore, the virtual flow $p_{i,j,t,s}^{virt}$ absorbs these inconsistencies and is bounded appropriately depending on the investment decision.

The use of virtual flows $p_{i,j,t,s}^{virt}$ therefore needs to be regulated. The virtual flows should be forced to zero when the line is installed, which can be done using a big-M approach as follows:

$$\left|p_{i,j,t,s}^{virt}\right| \leq \bar{p}_{i,j} \times yrfr_s \quad \forall\, t, s,\, ij \in \mathcal{L} \qquad \textit{eq. 11}$$

$$\bar{p}_{i,j} \leq M \cdot \bar{P}_{i,j}^{max} \cdot (1 - y_{i,j,t}) \quad \forall\, ij \in \mathcal{L} \qquad \textit{eq. 12}$$

In the first equation above (eq. 11), the flow in the virtual line is bounded by its virtual capacity $\bar{p}_{i,j}$ multiplied by *yrfr*$_s$ (the year fraction of timeslice *s*). (eq. 11). However, the maximum virtual capacity $\bar{p}_{i,j}$ is controlled in the second equation (eq. 12) by the (quasi-)binary[1] variable $y_{i,j,t}$, stating that it can be up to M times the maximum capacity of the candidate line $\bar{P}_{i,j}^{max}$ when the line is not installed, or zero when the line is installed ($y_{i,j,t} = 1$). The parameter $\bar{P}_{i,j}^{max}$ is the capacity bound for the candidate line (physical limit) and it is a required input parameter for all candidate lines.

The above constraints ensure that virtual flow is only allowed on candidate transmission lines that have not been built. The variables $y_{i,j,t}$ indicate whether line (i,j) is installed. If the line is installed ($y_{i,j,t} = 1$) the upper bound for the virtual flow becomes zero, forcing the virtual flow to be exactly zero. This mechanism corrects for the fact that all candidate lines are included in the PTDF matrix from the beginning, regardless of whether they are installed by neutralising their unintended influence unless explicitly activated. And if ($y_{i,j,t} = 0$), the real capacity and flow of any candidate line is bounded to zero.

In the MIP case, the quasi-binary variables $y_{i,j,t}$ are related to the internal binary variables VAR_DNCAP in TIMES for discrete new capacity investment (activated using the discrete capacity extension and NCAP_DISC). Therefore, the MIP implementation of the extension in TIMES utilises the discrete capacity extension of TIMES. The variables will then be genuinely binary whenever the capacity transfer fully matches with model periods, but otherwise $y_{i,j,t}$ may also have a value between 0 and 1 in the period of capacity retirement[2]. To avoid numerical issues and enforce stability in the optimisation, the relationship between the variables $y_{i,j,t}$ and the discrete capacity choices $dncap_{i,j,v}$ (i.e., the capacity installed for

[1] A quasi-binary variable is a continuous decision variable that behaves as if it were binary, even though it is not explicitly constrained to take only 0 or 1 values. Such a variable tends to take values close to 0 or 1 and rarely takes intermediate values.

[2] Usually, grid lines lifetime ranges from 30 to 60 years, which is much longer than the length of the periods usually modelled in TIMES; if sufficiently long lifetimes are provided together with a rational length of TIMES periods, then the variable will mostly take values very close to 0 or 1 and not intermediate values.

a line connecting nodes i and j in vintage v) can be established via the capacity transfer coefficients $\gamma_{v,t,i,j}$ that represents the remaining capacity (from 0 to 1) of the installed capacity $\bar{p}_{i,j}$ in year t. Note that only a single choice for the capacity size of an investment into a candidate line is allowed, but the timing of the investment may be fully left optimized.

$$y_{i,j,t} = \sum_{v} dncap_{i,j,v} \cdot \left(1 - \max\left(\frac{1-\gamma_{v,t,i,j}}{M}, \left(1-\gamma_{v,t,i,j}\right)^4\right)\right) \qquad \textit{eq. 13}$$

The expression under the exponent 4 ensures a smooth transition. The expression $\left(1-\gamma_{v,t,i,j}\right)^4$ as $\gamma \rightarrow 1$ and stays close to 1 when γ is small. This makes it a good approximation and offers numerical stability as it is flat near 1, instead of simply using $1-\gamma.$ In the LP case, the variable $y_{i,j,t}$ is defined in terms of new capacity variables:

$$y_{i,j,t} = 1 - \left(1 - \frac{\sum_v ncap_{i,j,v} \cdot \gamma_{v,t,i,j}}{\bar{P}_{i,j}^{max}}\right)/M \qquad \textit{eq. 14}$$

Equation (eq. 14) is a simple linear approximation of equation (13) by treating the variable $y_{i,j,t}$ as continuous in the interval [0,1], as the total capacity is bounded by $\bar{P}_{i,j}^{max}$.

2.2.2 Implementation in TIMES

In TIMES, pipelines are modelled as processes, which implies that there is a slight differentiation between the actual implementation and the above design. While the reader is directed to the User's Note (Lehtilä et al, 2025) for details, the table below provides the correspondence of the main variables, parameters and equations between the design and the TIMES implementation.

A. Mapping of Variables

Symbolic Name	GAMS Variable / Macro	Description
$g_{i,t}$, $d_{i,t}$	VAR_GRIDIO(r,t,c,n,s,io)	The variable representing the injection/withdrawal of grid-connected commodity c to/from grid node n in timeslice s.
$a_{k,t,s}^{VRT}$	VAR GNETINJ(r,t,n,s)	The artificial net injection variables in the PTDF formulation, for node n and timeslice s.
$p_{i,j,t,s}^{virt}$	VAR_GVIRT(r,t,p,n,s)	Virtual flows over candidate grid lines p for node n and timeslice s.
$\bar{p}_{i,j,t}$	VAR_XCAP(r,t,p)	Virtual capacity variable for the PTDF formulation with transmission expansion planning
$y_{i,j,t}$	VAR_PTDNCAP(r,t,p)	Quasi-binary variable controlling the installation of a virtual line, and representing the cumulative sum of VAR_DNCAP up to year t, where VAR_DNCAP in TIMES is the internal variable for discrete new capacity

B. Mapping of Parameters

Symbolic Name	GAMS Parameter	Meaning
$PTDF_{i,j,k}$	GR_PTDF(r,y,p,n,r2,n2)	Power transfer distribution factor of grid transmission line p in region r subject to node n2 in region r2

C. Mapping of Equations

Equation	GAMS Equation Name	Description / Meaning
eq. 10	EQ_GR_PTDFLO(r,t,p,n,s)	Linear DC power load flow equations for each grid pipeline – PTDF formulation.
eq. 11	EQ_GR_VIRTCP(r,t,p,n,s,bd)	Implied capacity requirement of virtual powerflow.
eq. 21	EQ_GR_VIRTBD(r,t,p)	Bound on the virtual powerflow capacity.

3. MODELLING OF THE GAS GRIDS

3.1 Philosophy of the Gas Grid Design in TIMES

The gas grid modelling extension in TIMES has been designed to enable users to represent the physical and operational characteristics of gas transport infrastructures in a manner that is consistent with the broader energy system model structure. The approach balances modelling fidelity with computational efficiency, making it suitable for large-scale, long-term scenario analysis while capturing essential spatial, temporal, and technical dynamics of gas networks.

3.1.1 Integrated Yet Modular Design

The gas grid module is structured as an optional and modular add-on to the standard TIMES Reference Energy System (RES). This means it can be included when needed—such as in studies involving gas infrastructure planning, decarbonisation of gaseous fuels, or gas-electricity integration—but it does not interfere with the modelling of other sectors when excluded. The gas infrastructure is embedded using the familiar TIMES modelling paradigm: processes represent pipelines and compressors, and commodities represent grid nodes, allowing users to work within a consistent conceptual framework.

This modularity also ensures flexibility, allowing users to model any number of nodes and pipelines, define arbitrary grid topologies (meshed or radial), and consider multiple types of gases (natural gas, biomethane, hydrogen, or CO_2) under a common set of modelling rules.

3.1.2 Physical Behaviour via Flow and Pressure Representation

TIMES gas grid modelling captures the physical characteristics of gas transport, including the dependence of flow on pressure gradients and the presence of intertemporal storage through linepack. Each node in the grid has a defined pressure range, and pipelines are characterised by their flow limits, frictional losses, and linepack capacity. Compressors, when included, can increase gas pressure between nodes. However, compressors are not represented as processes in the design, and any efficiency losses or costs related to the compressors should be included in the relevant attributes of the pipeline hosting the compressor.

To remain consistent with TIMES' optimisation framework, the non-linear physics of gas flow (notably the Weymouth equation) is approximated using linear methods. Users can select between a simplified linearisation (suitable for most planning scenarios) or a more detailed piecewise (but Mixed Integer Programming) approximation for higher accuracy.

3.1.3 Choice Between Unidirectional and Bidirectional Flow Representation

Users can choose to model pipelines as either unidirectional or bidirectional, depending on the desired trade-off between realism and computational tractability. The unidirectional option results in a fully linear (LP) formulation, which is well-suited to large-scale scenarios and simplifies interpretation. In contrast, bidirectional flow requires a mixed-integer linear programme (MILP), as it introduces binary variables to capture the directionality of flows. However, a MILP relaxation is also offered for bidirectional flows, which, in the tests conducted during the implementation of its design in TIMES, provided a very good approximation to the MILP. Unless the accuracy of MILP is required, it is recommended that most users enable the linear approximation of the unidirectional and bidirectional gas flows for computational efficiency.

The modelling of unidirectional and bidirectional flows enables the model to endogenously determine flow directions, thereby capturing flexibility benefits, such as reverse flow, in response to localised supply-demand shifts. Both approaches support linepack storage and compressor modelling. Still, the bidirectional formulation offers a more detailed and physically consistent representation of pipeline operation—particularly important in integrated electricity-gas system analyses.

3.1.4 Consistency with TIMES Data Structure

The gas grid module is fully integrated with TIMES' data structures and solution process. Inputs are provided using the same nomenclature and input formats as for other technologies, with additional tables to define node locations, pipeline connections, compressor specifications, and pressure settings. Where possible, variables such as inflows, outflows, and pressures are aligned with existing TIMES variables (e.g., those used for storage or power flow), supporting easier model development and scenario comparison.

3.1.5 Scalability and Application Scope

The design supports a wide range of applications—from the evolution of national gas systems under decarbonisation constraints to sector coupling assessments involving power-to-gas and hydrogen blending. Its compatibility with TIMES-VEDA workflows allows users to incorporate the gas grid module without disrupting existing calibration or reporting structures. Moreover, its flexibility in topology, gas type, and resolution accommodates both stylised studies and detailed infrastructure planning.

3.2 Fundamentals of Gas Flow Modelling (in TIMES)

The modelling of gas transport networks in TIMES is based on physical principles that govern steady-state gas flow in pipelines. The framework is designed to represent these principles in a simplified yet robust manner, enabling integration with long-term energy system planning. This section outlines the key concepts underpinning the modelling of gas flows, focusing on the factors driving gas movement, the structure of the network, and the underlying physics that are approximated in the model.

3.2.1 What Drives Gas Flow?

In gas networks, the movement of gas between nodes is governed by pressure differences. Gas naturally flows from areas of higher pressure to areas of lower pressure. This pressure differential is the fundamental driver of flow across the network. As the gas travels through a pipeline, frictional forces reduce pressure, resulting in a characteristic pressure drop from the inlet to the outlet. The model captures this effect by associating pressures with grid nodes and defining flow constraints that depend on the pressure difference between connected nodes.

3.2.2 Core Components of the Gas Network

The TIMES gas grid model considers two core physical components:

- **Pipelines**: These are used to transport gas between nodes. Each pipeline is modelled as a process, and its flow characteristics are determined by parameters such as diameter, length, and roughness. Pipelines also exhibit storage behaviour through linepack, which accounts for the temporary accumulation of gas due to compressibility.

- **Compressors**: These are specialised devices that increase the pressure of gas to maintain flow across long distances or through pipeline segments with high resistance. Compressors are modelled as separate processes that consume energy and apply a fixed or variable pressure increase. Unlike pipelines, gas in compressors flows from lower to higher pressure nodes.

Together, these components define the physical layout and operational behaviour of the gas grid. In TIMES, pipelines are mapped to the RES structure as distinct processes with input and output links to pressure-defined nodes. Compressors are not TIMES processes, but they are introduced into the grid topology in a simplified representation, suitable for modelling their behaviour, but it neglects any energy consumption associated with the change of pressure induced by the compression – this energy consumption should be entered as an efficiency loss in the pipeline process.

3.2.3 Key Characteristics of the Pipelines and the Weymouth Equation

The behaviour of a gas pipeline is shaped by several technical attributes, which influence its capacity and pressure drop characteristics:

- The **inner diameter** and **length** of the pipeline determine the frictional resistance and, thus, the pressure loss along the pipeline.

- Gas composition and **density**, along with **temperature**, affect the compressibility of the gas and the resulting flow rate.

- **Linepack**, the gas stored within the pipeline, is proportional to the average pressure between the two ends and the physical volume of the pipeline. It enables short-term flexibility by decoupling inflow and outflow over time.

These characteristics are summarised in a simplified formulation, allowing TIMES to approximate the effects of pipeline physics on system-wide gas flows and storage dynamics.

3.2.4 Gas Flow Modelling

In pipeline systems, gas flows from high to low pressure, and the flow rate depends on:

- The pressure drop between upstream and downstream nodes

- The physical characteristics of the pipeline (diameter, length, friction)

- The gas composition and temperature

The central physical relationship governing steady-state gas flow in pipelines is the Weymouth equation. This non-linear equation expresses the squared gas flow as proportional to the difference between the squares of the inlet and outlet pressures, accounting for pipeline resistance and gas properties. It is widely used in transmission-level gas system models. The Weymouth equation is a standard non-linear expression used to model steady-state gas flow in pipelines. It relates the volumetric flow rate of gas to the pressure difference between two nodes in the gas network. In its most common form, the equation is given as:

$$q_{m,n,t} = \frac{(C \cdot D^{2.666} \cdot E)}{(L \cdot \sqrt{G \cdot z \cdot T})} \cdot \frac{T_{sc}}{P_{sc}} \cdot \sqrt{p_{m,t}^2 - p_{n,t}^2} = W_{m,n} \cdot \sqrt{p_{m,t}^2 - p_{n,t}^2} \qquad \textit{eq. 15}$$

In the above formulation:

$q_{m,n}$: gas flow rate between two nodes m, n and time t at standard conditions Sm^3/d (SFC/d)

C = 1.162·10^7 in metric units (433.5 in English units)

p_m : pressure at node m in kPa (psia)

p_n : pressure at node n in kPa (psia)

P_{sc} : standard pressure in kPa (psia)

T_{sc} : standard temperature in K (R)

T : mean temperature in K (R)

D : inner pipe diameter in m (in)

L : pipeline length in m (mile)

G : relative gas density

z : mean compressibility factor

E : pipeline efficiency

$W_{m,n}$: Weymouth constant

In the above equation and in all equations followed, t represents a timeslice and we ignore the year index for simplicity in the formulation.

With a re-arrangement of the terms of the equation (eq. 15), we obtain a much simpler formulation where all pipeline and gas-specific terms have been aggregated into a constant called the *Weymouth constant*, which is represented by $W_{m,n}$. When characterising a pipeline, the Weymouth constant is sufficient, but if it is not available, then the user should calculate all the above parameters entering into the constant.

However, because the Weymouth equation is non-convex and quadratic, it cannot be directly incorporated into the linear programming structure of TIMES. Instead, the model uses linear approximations, such as Taylor series expansions or piecewise linear steps, to replicate its behaviour within solvable LP or MILP formulations. These approximations enable TIMES to capture the influence of pressure gradients on gas flows and to enforce realistic limits on pipeline utilisation and flow direction.

Each node's pressure must lie within operational limits to prevent unrealistic pressure values and ensure safe transmission:

$$p_n^{min} \leq p_{n,t} \leq p_n^{max} \quad \textit{eq. 16}$$

When compressors are present, they increase downstream pressure. Assuming that p_m represents the pressure at the downstream node m, and $p_{k,m}^{add}$ represents the pressure boost at the inlet of pipeline k at node m, then the compressor boost equation can be written as follows:

$$p_{k,m,t}^{add} \leq (\Gamma_{k,m} - 1) \cdot p_{m,t} \quad \textit{eq. 17}$$

The parameter $\Gamma_{k,m} \geq 1$ is the compressor pressure ratio or boost factor. The above equation imposes an upper bound on the pressure boost that can be achieved by the given compression factor. It follows that when a compressor is absent, the boost factor can be set to 1. While the above equation ensures a maximum pressure lift, it does not model compressor energy consumption unless a separate process models the energy use.

In this regard, in the presence of a compressor, the pipeline inlet pressures (after any compressor, i.e. the outlet pressure of a compressor when present) are actually:

$$\hat{p}_{k,m,t} = p_{m,t} + p_{k,m,t}^{add} \quad \textit{eq. 18}$$

The above equation defines the pressures that will enter into the linearised version of the Weymouth equation (see section 3.2.4.1).

In high-pressure natural gas transmission systems, the volume of gas stored within pipelines—referred to as linepack—plays a critical role in short-term system flexibility. By increasing the pressure within a pipeline, it is possible to store additional gas without the need for separate storage infrastructure. This distributed storage capability enables the gas system to absorb or release gas across time periods, helping to manage temporal fluctuations in supply and demand, particularly at daily or sub-daily resolution.

The linepack is modelled as a time-dependent storage state within each pipeline. The basic formulation tracks the linepack level $h_{m,n,t}$ in the pipeline between node n across consecutive time periods:

$$h_{m,n,t} = h_{m,n,t-1} + q_{m,n,t}^{in} - q_{m,n,t}^{out} \quad \textit{eq. 19}$$

Where $q^{in}_{m,n,t}$ is the gas entering the pipeline from node m at time t, while $q^{out}_{m,n,t}$ is the gas exiting the pipeline to node n at time t. It is assumed that when linepack is modelled the gas flow variable $q_{m,n,t}$ that enters the Weymouth and pipeline capacity constraints is defined as the average of the inflow and outflow of the pipeline process in each time slice. This mid-flow formulation allows consistent pressure-flow modelling while capturing the effect of linepack dynamics through the decoupling of injection and withdrawal flows:

$$q_{m,n,t} = \frac{1}{2}\left(q^{in}_{m,n,t} + q^{out}_{m,n,t}\right) \qquad \textit{eq. 20}$$

In addition, to link the amount of gas stored in the pipeline to its pressure, a linear pressure-inventory relationship can be enforced between the pressures $p_{m,t}$, $p_{n,t}$ and a linepack pressure coefficient for the pipeline between nodes m and n. This relationship allows pressures at the nodes to influence how much gas is held in the pipeline dynamically

$$h_{m,n,t} = \beta_{m,n} \frac{p_{n,t}+p_{m,t}}{2} \qquad \textit{eq. 21}$$

The coefficient $\beta_{m,n}$ captures the physical characteristics of the pipeline and the thermodynamic properties of the gas. It presents the amount of gas (in mass or energy terms) that can be stored per unit of average pressure. Formally, $\beta_{m,n}$ can be derived from the ideal gas law and pipeline geometry as follows:

$$\beta_{m,n} = \frac{V_{m,n} \cdot \rho_{ref}}{P_{ref}} \qquad \textit{eq. 22}$$

Where $V_{m,n}$ is the internal volume of the pipeline between nodes m and n, ρ_{ref} is the reference gas density and P_{ref} is the reference pressure for scaling. In practice $\beta_{m,n}$ is a user supplied parameter that defines the maximimum linepack level as a function of pressure. Larger values indicate pipelines that offer greater flexibility through pressure variation.

In the presence of a linepack, a minimum linepack level may be needed to guarantee physical feasibility. Therefore, the variable $h_{m,n,t}$ can be defined to have a minimum value $H^0_{m,n,t}$. Defining such a minimum level may ensure that the pipeline contains a realistic and usable quantity of gas under compression.

$$h_{m,n,t} \geq H^0_{m,n,t} \qquad \textit{eq. 23}$$

In the above, $H^0_{m,n,t}$, however, should not be set higher than the average value of the pressure range at node m would imply, to prevent any infeasibility due to imposing linepack at an overly high level (compare to eq. 21). Therefore, if set too high, $H^0_{m,n,t}$ is down-adjusted.

In gas network modelling, each pipeline has a finite capacity to transport gas due to physical limitations such as pipe diameter, allowable pressure differences, and frictional effects.

$$\left|q_{m,n,t}\right| \leq Q^{max}_{m,n,t} = C_{m,n} \cdot \theta_{m,n,t} \qquad \textit{eq. 24}$$

In the above equation, the $q_{m,n,t}$ is the average of the inflow and outflow of the pipeline process, and $Q^{max}_{m,n,t}$ is the maximum permissible flow capacity calculated endogenously. This is based on the installed capacity $C_{m,n}$ multiplied by the time slice availability factor $\theta_{m,n,t}$. The absolute notation in the above equation is used to capture bidirectional flows.

3.2.4.1 Linearisation of the Weymouth equation

The Weymouth equation, in its standard non-linear form, relates the gas flow between tow nodes to be the square root of the difference in squared pressures. This non-linear formulation poses challenges for linear programming based energy system models such TIMES. To maintain LP structure, two linearization approaches are offered, each balancing computational tractability and physical fidelity.

A) Taylor Approximation

The Weymouth equation can be approximated via a first-order Taylor expansion, linearising the square root function around a reference pressure pair. For the linearization, the following auxiliary variable is defined for each pipeline *k* from node *m* to node *n*:

$$\Delta p_{k,m,n,t} = \hat{p}_{k,m,t} - \hat{p}_{k,n,t} + \Delta p_{k,n,m,t} \qquad \textit{eq. 25}$$

The above variable $\Delta p_{k,m,n,t}$, subject to non-negativity constraint, plays three critical roles in the approximation:

i) enforces non-negative flows: since flow in gas pipelines physically occurs from higher to lower pressure, $\Delta p_{k,m,n,t}$, helps preserve this directionality in the linearised context by compensating for cases where the pressure difference alone might not ensure a positive flow. It acts as a "buffer" to prevent infeasible or reversed flows within the LP structure.

ii) allows bi-directional modelling: since the TIMES extension also supports bi-directional flows (see section 3.2.4.4), to maintain consistency $\Delta p_{k,m,n,t}$ is coupled with its reverse via the above equation, ensuring that only one direction can be active at a time, and maintains feasibility and balance when flows switch direction.

iii) maintaining LP compatibility: in the bi-directional case, the variable $\Delta p_{k,m,n,t}$ is introduced into the Weymouth approximation in the purpose of replacing the big-M term, to provide an LP relaxation and thus extend the applicability of the equation into the LP domain.

Having defined the pressures after the boost and the pressure difference variables, the linearised formulation of the Weymouth equation for pipeline *k* can be given as follows:

$$q_{k,m,n,t} \leq W_{k,m,n} \left[\frac{\tilde{p}_{m,\omega}}{\sqrt{\tilde{p}^2{}_{m,\omega} - \tilde{p}^2{}_{,n,\omega}}} \cdot \hat{p}_{k,m,t} - \frac{\tilde{p}_{n,\omega}}{\sqrt{\tilde{p}^2{}_{m,\omega} - \tilde{p}^2{}_{n,\omega}}} \cdot \hat{p}_{k,n,t} \right] \qquad \textit{eq. 26}$$

As stated before, the linearisation of the Weymouth equation is performed by taking a finite number of samples $\omega \in \Omega$ around a reference point. The literature (e.g. Shin et al. 2021) suggests several key considerations when selecting the pressure pairs $\tilde{p}_{m,\omega}$ and $\tilde{p}_{n,\omega}$:

a) Feasibility constraints: to ensure that the linearisation remains within the physically meaningful domain, the sampled pressures must satisfy $\tilde{p}_{m,\omega} \geq \tilde{p}_{n,\omega}$, $\tilde{p}_{m,\omega} \in [p_m^{min}, p_m^{max}]$, $\tilde{p}_{n,\omega} \in [p_n^{min}, p_n^{max}]$. This avoids sampling in non-physical or infeasible regions of the square root function

b) Representation of expected operating conditions: the sampling should reflect typical operating pressures in the network. Reference points may be chosen around historical or anticipated pressure levels, especially in pipelines with high utilisation. This improves the fidelity of the linear approximation where it matters most

c) Number of linearisation points: multiple pairs allow for a piecewise approximation that better captures curvature. Usually, $|\Omega| = 20$ is enough for an accurate approximation.

d) Symmetry for bidirectional flow: for pipelines that allow bidirectional flows, symmetric sampling is recommended to ensure consistent treatment of flows in both directions and prevent the LP from biasing one direction due to linearisation artefacts.

e) Impact assessment: sensitivity analysis should be used to determine whether the chose sampling introduces significant error in marginal cost estimations or feasibility. In such cases, pressure pair selection can be refined, or the number of reference points increased.

B) Stepwise Approximation

The TIMES implementation offers also a stepwise approximation. This approach represents the nonlinear pressure-flow relationship using a convex combination of predefined flow segments. It enables accurate representation of nonlinear behaviour under MIP as well as an LP-compatible formulation, thereby avoiding the necessity of introducing integer of binary variables when including bi-directional pipelines.

The alternative formulation divides the pressure difference into a linear combination over a number of predefined pressure-difference variables, for which the contribution the Weymouth equation can be pre-calculated. For this, a cartesian product of fixed pressure points on both sides, from higher to lower pressures in each direction is defined, and the total value of the Weymouth function is obtained as the sum of Weymouth contribution over these stepped pressure-difference variables. In this regard, the Weymouth equation and the associated pressure constraints can be written as:

$$q_{k,m,n,t} \leq W_{k,m,n} \cdot \sum_{\omega \in \Omega_{k,m,n}} \lambda_{\omega,t} \cdot \phi_{\omega} \qquad \textit{eq. 27}$$

$$\begin{aligned} \hat{p}_{k,m,t} &= \textstyle\sum_{\omega \in \Omega_{k,m,n}} \lambda_{\omega,t} \cdot \tilde{p}_{k,m,\omega} + \sum_{\omega \in \Omega_{k,n,m}} \lambda_{\omega,t} \cdot \tilde{p}_{k,m,\omega} \\ \hat{p}_{k,n,t} &= \textstyle\sum_{\omega \in \Omega_{k,m,n}} \lambda_{\omega,t} \cdot \tilde{p}_{k,n,\omega} + \sum_{\omega \in \Omega_{k,n,m}} \lambda_{\omega,t} \cdot \tilde{p}_{k,n,\omega} \end{aligned} \qquad \textit{eq. 28}$$

Where $\lambda_{\omega,t}$ is the weight of the segment ω in timeslice t in the Weymouth equation, ϕ_{ω} is the precomputed pressure-flow coefficient for segment ω, $\Omega_{m,n}$ is the set of pressure segment pairs from node m to node n.

For the LP case, the following constraint is additionally imposed to improve the quality of the LP relaxation for bi-directional pipelines k:

$$\sum_{\omega \in \Omega_{k,m,n}} \lambda_{\omega,t} + \sum_{\omega \in \Omega_{k,n,m}} \lambda_{\omega,t} \leq 1 \qquad \textit{eq. 29}$$

The above condition attempts at restricting flows to be active only in one direction at a time across the pipeline, but it will by no means guarantee such. Under LP, good accuracy of the approximation can only be expected for unidirectional grids. The stepwise function remains a convex combination, meaning that flow is interpolated between samples pressure pairs without violating linearity. In addition, the solution can accommodate zero flow (i.e., all $\lambda_{\omega,t} = 0$ is allowed). And, importantly, the flow direction switching is well handled when using binary variables.

Here is worthy to note that unlike the stepwise approximation, the Taylor linearisation of the Weymouth equation in TIMES does not use explicit pressure pair sampling over $\Omega_{m,n}$ and $\Omega_{n,m}$. Instead, it applies a first-order approximation at chosen reference points (defined by the ω index), with directionality embedded in the coefficients. For each link, only one direction is active at a time, and the pressure boost term accounts for the compressor if present.

3.2.4.2 Sampling strategies for linearising the Weymouth Equation

a) Taylor approximation

In the Taylor linearisation approach, the non-linear Weymouth equation is approximated by performing first-order Taylor expansions around several reference pressure pairs (p_m*, p_n*). Each pair defines a local linear constraint, and the collection of these constraints provides a convex approximation of the original non-linear relationship. Literature suggests using 10–20 reference points to ensure sufficient coverage of the operational pressure range (see Shin et al., 2022). For example, assuming a pressure range of 40–70 bar at node m and 25–60 bar at node n, 20 Taylor expansions may be defined across this space, such as: 45, 30), (47, 32), ..., (65, 50) — each producing a separate linear constraint based on the Taylor expansion of the Weymouth formula.

It is important to note here that the Taylor approximation is sensitive not only to the number of points but also to how these points are selected. Having a sufficient number of good-quality pressure points, the violations of the Weymouth equation (as a result of its linearisation) can be made small, at the expense of the solution time. We have found that usually more than 30 points for each pipeline direction (i.e., 50% more than what the literature suggests) can limit violations.

b) Stepwise approximation

The stepwise linearisation discretises the Weymouth curve into a small set of flow-pressure sample points. Each point corresponds to a flow value pre-computed via the full non-linear equation, and a TIMES lambda variable selects the convex combination of these flows to approximate the actual flow. Typical implementations use 3 to 7 such steps to avoid excessive LP size. For example, sampling 5 points over a Δp range of 10–25 bar might yield: (45, 35), (50, 35), (55, 40), (60, 45), (65, 50) — with corresponding flow values calculated using the Weymouth equation offline.

In our tests, this alternate formulation resulted in potential benefits: 1) it proved not to cause significant violations of the tree non-linear Weymouth equations, 2) it avoided the whole issue of generating a sufficiently good sample of fixed pressure points, and 3) it provided a well-working and easy-to-use LP relaxation for unidirectional and even reasonably well-working for bi-directional networks.

3.2.4.3 Comparison of sampling strategies

Feature	Taylor Approximation	Stepwise Approximation
Typical number of points	10–20	3–7
Scope	Local linearisation at reference points	Global approximation via convex sum
Constraint form	One inequality per Taylor point	Convex combination constraint
LP complexity	Grows linearly with OMG points	Grows with # of steps × arcs
Accuracy	High (with sufficient expansions)	High (with strategic sampling)

To help the TIMES user, we have also implemented an *auto-generation* procedure for the two sampling strategies. It is based on sampling from the high-pressure to the low-pressure side of the pipeline by gradually increasing the pressure drops in steps. Hence, starting from the maximum pressure on the high-pressure side $p_{m,t} \cdot \max(1, \Gamma_{k,m})$, the first pressure drop d(1) is set to be 1/1000 of it and all subsequent drops d(n) are always d(n)=1.2 x d(n-1). The number of steps n is dependent on the maximum drop, such that the last step should exceed the maximum drop to cover the whole space. Only the proportional pressure drops in fact matter here, and therefore, this set of points, based on the pressure drops from the maximum pressure, is sufficient.

Please see the annex for an illustration of the linearisation for the Weymouth equation with auto-generated pairs of pressure points.

3.2.4.4 Bi-directional flows

The modelling of bi-directional flows increases the accuracy of the gas grid representation, together with the complexity and computational resources required to solve the model. This formulation allows the gas flow $q_{k,m,n,t}$ to be either positive, or zero or negative, depending on the sign of the pressure difference.

For the bi-directional flow modelling, a binary variable $y_{k,m,n,t}$ needs to be introduced, which defines the direction of the flow in pipeline k. The following constraints enforce that only one direction, $m \rightarrow n$ or $n \rightarrow m$ will be active in each timeslice

$q_{k,m,n,t} \leq M \cdot y_{k,m,n,t} \quad , \quad q_{k,n,m,t} \leq M \cdot (1 - y_{k,m,n,t})$	*eq. 30*
$\Delta p_{k,m,n,t} \leq M \cdot y_{k,m,n,t} \quad , \quad \Delta p_{k,n,m,t} \leq M \cdot (1 - y_{k,m,n,t})$	*eq. 31*

When modelling bi-directional flows, and based on the above, the Weymouth approximation needs to be reformulated according to the following constraints:

$q_{k,m,n,t} \leq W_{k,m,n} \left[\frac{\tilde{p}_{m,\omega}}{\sqrt{\tilde{p}^2{}_{m,\omega} - \tilde{p}^2{}_{n,\omega}}} \cdot \hat{p}_{k,m,t} - \frac{\tilde{p}_{n,\omega}}{\sqrt{\tilde{p}^2{}_{m,\omega} - \tilde{p}^2{}_{n,\omega}}} \cdot \hat{p}_{k,n,t} \right] + M \cdot (1 - y_{k,m,n,t})$	*eq. 32*

$q_{k,n,m,t} \leq W_{k,m,n} \left[\frac{\tilde{p}_{k,n,\omega}}{\sqrt{\tilde{p}^2{}_{n,\omega} - \tilde{p}^2{}_{m,\omega}}} \cdot \hat{p}_{k,n,t} - \frac{\tilde{p}_{k,m,\omega}}{\sqrt{\tilde{p}^2{}_{n,\omega} - \tilde{p}^2{}_{m,\omega}}} \cdot \hat{p}_{k,m,t} \right] + M \cdot y_{k,m,n,t}$	*eq. 33*

The big-M terms on the right, involving the binary variables, can be given a good estimates by using the relation for $\Delta p_{k,m,n,t}$ (eq. 31) and thereby retaining full LP-compatibility:

$$q_{k,m,n,t} \leq W_{k,m,n} \left[\frac{\tilde{p}_{m,\omega}}{\sqrt{\tilde{p}^2{}_{m,\omega} - \tilde{p}^2{}_{n,\omega}}} \cdot (\hat{p}_{k,m,t} + \Delta p_{k,n,m,t}) - \frac{\tilde{p}_{n,\omega}}{\sqrt{\tilde{p}^2{}_{m,\omega} - \tilde{p}^2{}_{n,\omega}}} \cdot \hat{p}_{k,n,t} \right] \quad \text{eq. 34}$$

$$q_{k,n,m,t} \leq W_{k,m,n} \left[\frac{\tilde{p}_{n,\omega}}{\sqrt{\tilde{p}^2{}_{n,\omega} - \tilde{p}^2{}_{m,\omega}}} \cdot (\hat{p}_{k,n,t} + \Delta p_{k,m,n,t}) - \frac{\tilde{p}_{m,\omega}}{\sqrt{\tilde{p}^2{}_{n,\omega} - \tilde{p}^2{}_{m,\omega}}} \cdot \hat{p}_{k,m,t} \right] \quad \text{eq. 35}$$

The stepped Weymouth approximation allows modelling bi-directional flows also without binary variables, but with risk of the quality of the solution being more or less compromised.

3.2.4.5 Pipelines with multiple gases flowing through them

In many real-world gas infrastructures, pipelines are used to transport more than one type of gas. These can include blends of natural gas, biomethane, hydrogen, synthetic methane, or even non-energy gases. A flexible structure to model such pipelines by tracking the flow of individual gas carriers, while maintaining an aggregated treatment of the physical pressure dynamics for pipeline constraints such as the Weymouth equation or linepack is provided.

This implies that the gas flow variables are separate for each gas, but the Weymouth and linepack constants refer to the representative gas flowing in the pipeline. Both constants $W_{k,m,n,t}$ and $\beta_{m,n,t}$ are now time-dependent and gas-dependent and can be defined for the typically the dominant or most physically relevant gas for which the pressure-flow dynamics are explicitly modelled. The variable $q_{k,m,n,t}$ in this regard reflects the reflects the total aggregated flow in the pipeline k, which is driven by the sum of flows of individual gases.

As an example, in a natural gas pipeline where hydrogen is also injected, the Weymouth and linepack constants can be defined to represent the physical properties of the natural gas. In a similar analogy, if a pipeline transfers two gases A and B and up to time t gas A is the dominant, but after time t gas B becomes the representative gas in the pipeline, then the Weymouth and linapack constants can be defined to reflect the physical properties of gas A until time t and the physical properties of gas B after time t.

This design enables modellers to retain carrier-specific detail for emissions accounting, blending rules, or energy system interactions, while reducing complexity in the pressure modelling by avoiding carrier-specific Weymouth equations. However, it is important to select the representative carrier wisely and ensure that its physical characteristics (e.g., density, pressure bounds) are broadly consistent with the blended or shared operation of the pipeline.

Box 1: Approaches to Multi-Carrier Gas Pipeline Modelling – a reflection

In modelling shared pipeline infrastructure across multiple energy carriers (e.g. CH_4, H_2, CO_2), several approaches can be considered. The option implemented in TIMES models each carrier with separate flow variables and energy balances, while all pipeline physical constraints — such as the Weymouth equation and linepack — refer to a representative carrier (typically methane) that defines the pipeline's technical characteristics. This approach offers a transparent physical formulation and allows full carrier-specific tracking, while simplifying infrastructure representation. A second option is to define a shared pipeline commodity (e.g. PIPEFLO) that aggregates flows from all carriers via commodity conversion processes; this keeps the model compact and avoids duplicated structures but requires artificial zero-loss mappings and loses detail on carrier-specific physics. A third option involves modelling each carrier along its own dedicated pipeline process chain, with a mutual exclusivity constraint that restricts use to a single carrier at any time. While this ensures accurate carrier-specific physics, it introduces binary variables and complicates the optimisation. Finally, a theoretically flexible but technically heavier option is to apply explicit multi-commodity flow coupling, where flows of different gases are normalised via energy or volumetric factors and constrained against shared capacity. Although this captures physical sharing most directly, it complicates the formulation of pressure and storage equations and may hinder solver performance. Overall, the TIMES implementation strikes a pragmatic balance between carrier-specific tracking and infrastructure simplification, with modest assumptions around the dominance or representativeness of the physical gas type.

3.2.5 Implementation in TIMES

In TIMES pipelines are modelled as processes, which implies that there is a slight differentiation between the actual implementation and the above design. While the reader is directed to the User's Note (Lehtilä et al, 2025) for details, the table below provides the correspondence of the main variables, parameters and equations between the design and the TIMES implementation.

A. Mapping of Variables

Symbolic Name	GAMS Variable / Macro	Description
$q_{k,m,n,t}$	VAR_GG_MF(r,t,p,c,reg,com,s)	Aggregated mass flow over all gases in a pipeline k between nodes m and n
$p_{m,t}$	VAR_GG_PR(r,t,c,s)	Pressure at node m (commodity c) and time t, timeslice t
$p^{add}_{k,m,t}$	VAR_GG_PADD(r,t,p,c,s)	Pressure boost added by compressor at pipeline k, node m.
$\hat{p}_{k,m,t}$	VAR_GG_PRIO(r,t,p,c,s)	Pressure at compressor outlet of pipeline k, sum of nodal pressure and pressure boost.
$\Delta p_{k,m,n,t}$	VAR_GG_PDIFF(reg,t,p,com,r,c,s)	Auxiliary pressure difference over pipeline k used in linearised flow constraints.
$h_{m,n,t}$	VAR_GG_HLIP(r,t,p,s)	Linepack in line between nodes m and n
$\lambda_{\omega,t}$	VAR_GG_STEP(r,t,p,c,s,J,JJ)	Weight variable for each segment in stepwise Weymouth approximation.
$y_{k,m,n,t}$	VAR_GG_Y(r,t,p,m,r2,u,ts)	Binary variable for the gas flow direction for pipeline k between nodes m and n

B. Mapping of Parameters

Symbolic Name	GAMS Parameter	Meaning
$W_{k,m,n,t}$	GG_KGF(r,t,p,c)	Weymouth coefficient of pipeline p for gas c
$\beta_{m,n}$	GG_KLP(r,t,p,c)	Linepack constant for gas c
$\Gamma_{k,m}$	GG_GAMMA(r,t,p,c)	Compressor factor for pipeline p at node m
p_m^{min}	GG_PRBD(r,t,c,'LO')	Lower bound on pressure at node m
p_m^{max}	GG_PRBD(r,t,c,'UP')	Upper bound on pressure at node m
G	GG_DENS(R,C)	Physical density of the gas carrier used for mass/volume conversions.
$\tilde{p}_{m,\omega}$	GG_PP(r,y,p,n,bd,omg)	Fixed pressure points for node n in the Weymouth Taylor expansion
ϕ_ω	GG_PPW(...)	Pressure sample points for stepwise approximation of Weymouth.
M	GG_MM(r,t,p,c)	Large constant used to relax flow constraints under certain conditions.

C. Mapping of Equations

Equation	GAMS Equation Name	Description / Meaning
eq. 17	EQ_GG_GAMA(r,t,p,m,r2,u,ts)	Pressure boost defining equation
eq. 19	EQ_GG_HLIP(r,t,p,m,r2,u,ts)	Line-pack storage balance equation
eq. 21	EQ_GG_HLEV(r,t,p,m,r2,u,ts)	Defining equations for line-pack levels
eq. 25	EQ_GG_PDIF2(r,t,p,m,r2,u,ts)	Pressure difference defining equation
eq. 26	EQ_GG_WEYMTX(r,t,p,m,r2,u,ts,o)	Weymouth equation with Taylor expansion
eq. 27	EQ_GG_WEYMST(r,t,p,m,r2,u,ts)	Weymouth equation with stepped linearization
eq. 30	EQ_GG_MBND(r,t,p,m,r2,u,ts)	Mass flow bounds in bi-directional case
eq. 31	EQ_GG_PDIF1(r,t,p,m,r2,u,ts)	Pressure difference bounds in bi-directional case

3.3 Other Grid Modelling Features

The current extension does not cancel other grid features that already supported by TIMES. These include:

- Allocation of Generation and Load to Grid Nodes
- Simplified N-1 Security Constraints
- Costs on Grid Flows

Below we shortly elaborate on these features, and the user is directed to the *User Note* for more details on how to enable these features and use them in TIMES.

3.3.1 Allocation of Generation and Load to Grid Nodes

The description of the electricity and gas grids can be implemented as an add-on, without changing the original structure of the model. In this regard, grid injections and grid withdrawals need to be determined by node. This is achieved by linear allocation equations of electricity or gas supply and demand to the nodes via exogenous shares for this allocation. While this approach does not change the original structure of the model, it entails the limitation that the injection and withdrawal shares are pre-defined. For an explanation on how to use the add-on grid modelling in TIMES, the user is directed to the *User Note* accompanying this report.

An alternative approach is to use an in-place grid structure. However, this will change the reference energy system of the original TIMES instance, as it will require the definition of supply and demand processes per node. In this approach, the output of the supply process or the input to the demand process is the node, which is represented as commodity in TIMES.

An example of in-place grid structure in TIMES is provided in https://forum.kanors-emr.org/showthread.php?tid=221

3.3.2 Simplified N-1 Security Constraints

For N-1 security considerations for grids involving energy commodities, we may want to derate the maximum capacity of a line or pipeline by a given percentage. For the electricity grids this is achieved via a dedicated parameter GR_XBND that denotes the percentage of the available capacity after considering N-1 security constraints.

For the gas grids, an equivalent constraint can be introduced via a user constraint. In the gas grids the user needs to refer to the VAR_IRE variables instead of VAR_GRIDIO for getting the net injection to be bounded in each region. Please consult the *User Note* for more details.

3.3.3 Costs on Grid Flows

The TIMES implementation includes a parameter COM_CSTBAL which enters into the objective function and can be used for defining cost coefficients for the different grid flows (electricity or gaseous commodities and emissions). The parameter includes a cost type index dimension, which allows to define costs for the supply (type=PRD), imports (type=IMP), exports (type=EXP), net impots (type=NTX), consumption (type=CON) or net positive generation (type=NPG). The user is directed to the *User Note* for more details.

4. APPLICATION OF THE EXTENSION

For detailed instructions on how to enable and use the extension in TIMES, please refer to the *User Note* and the accompanying *demo TIMES models* that implement the extension in VEDA wit detail instructions and illustrations.

5. ANNEX

Illustration of the linearization for the Weymouth equation with auto-generated pairs of pressure points.

Figure 1. Linearization of the Weymouth flow bounds – example with a max. pressure drop of 35% from the inlet pressure.

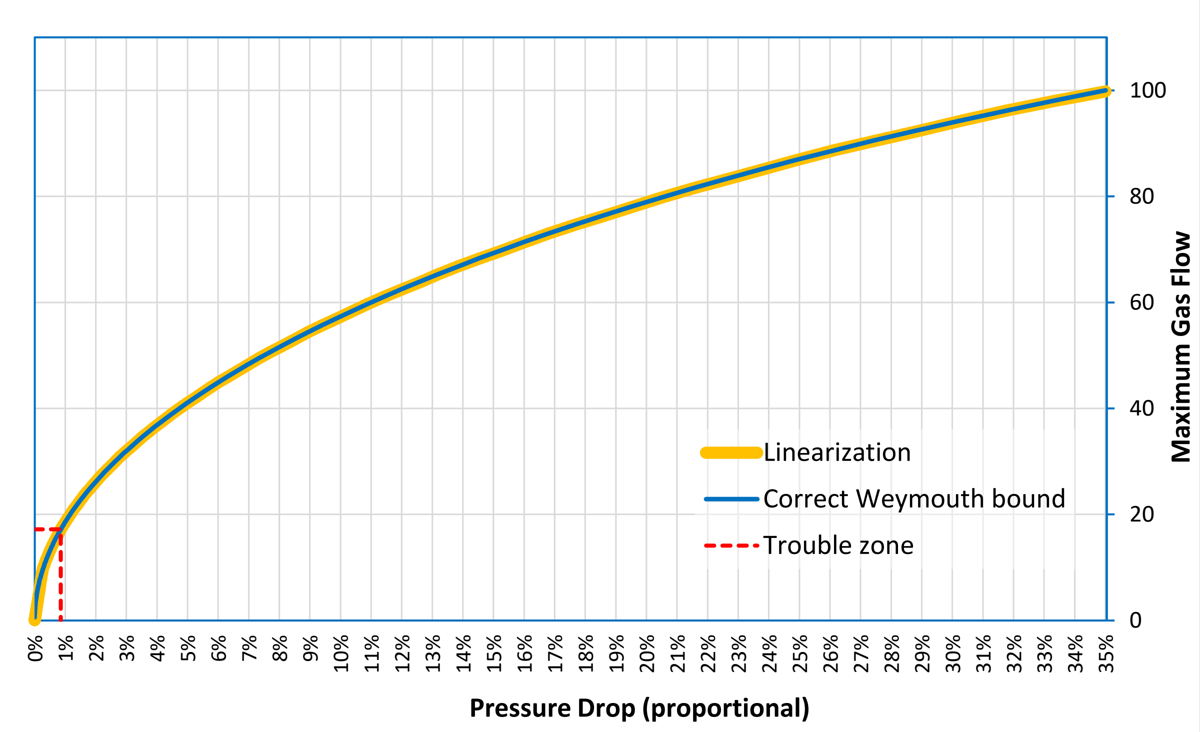


Figure 2. Linearization of the Weymouth flow bounds – logarithmic scales.

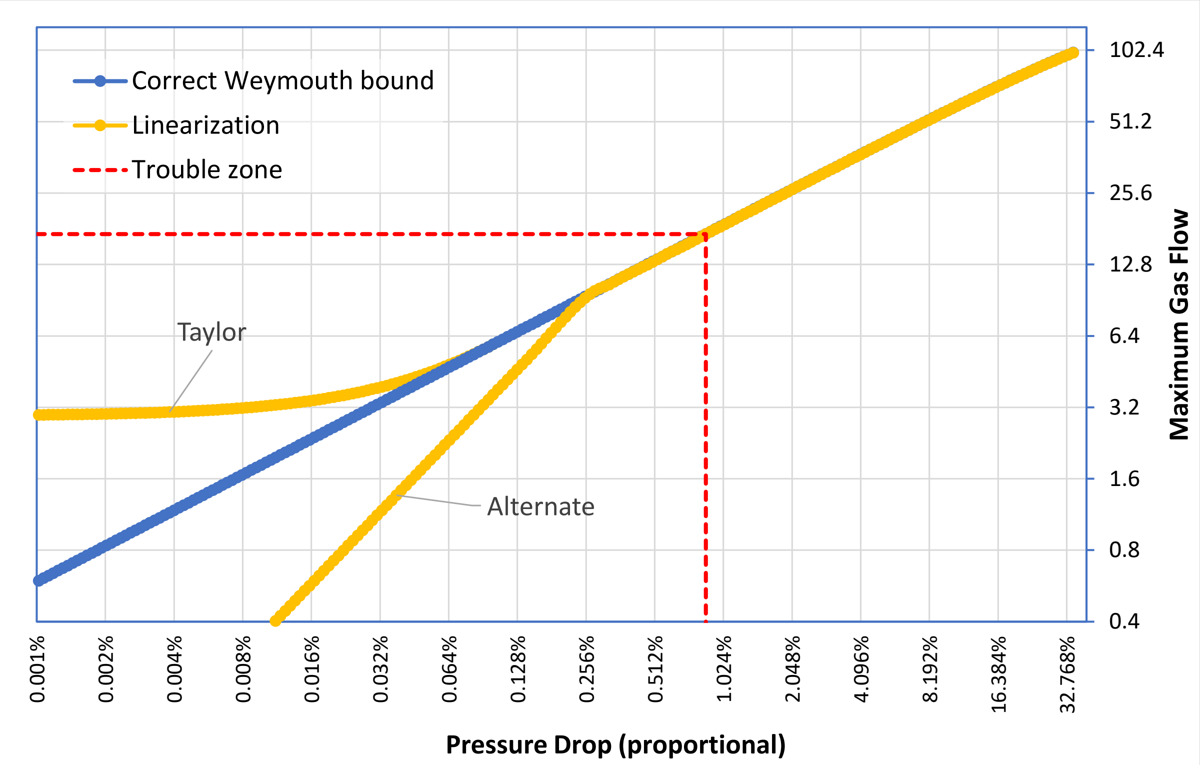
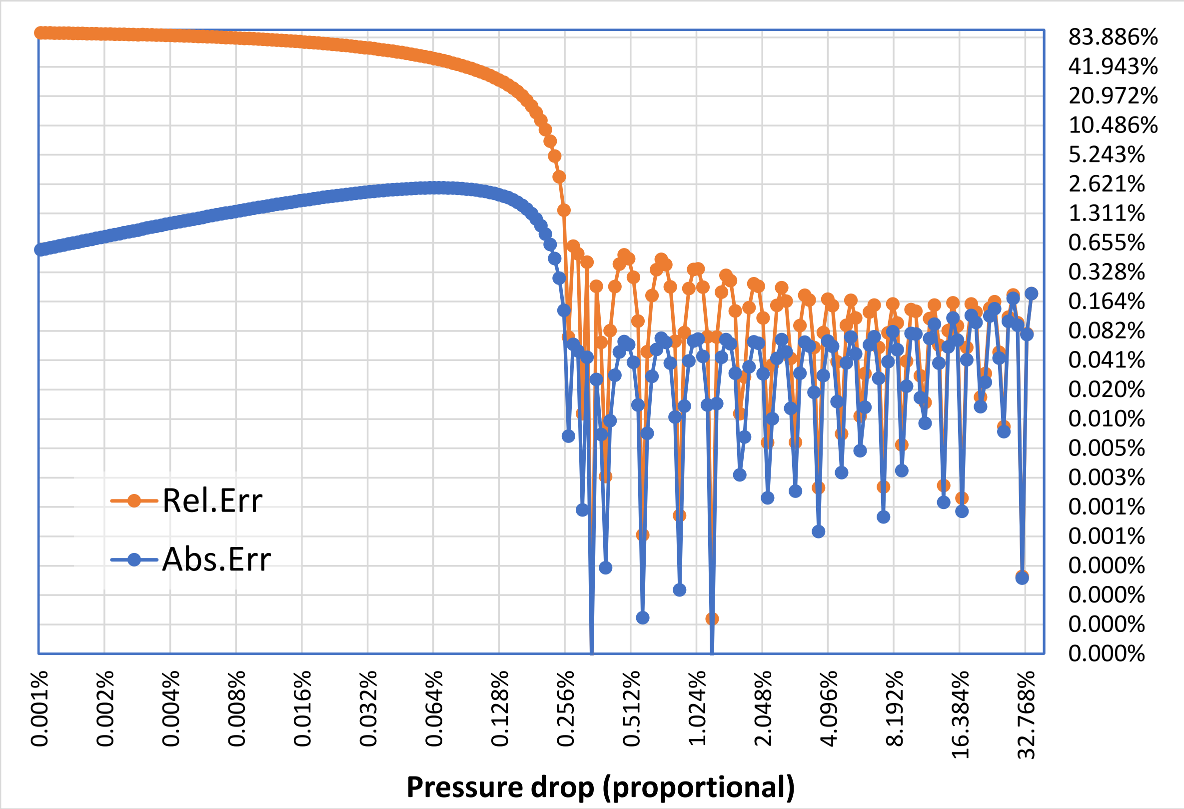


Figure 3. Errors in the linearization of the Weymouth flow bounds – alternate formulation. Errors remain very small over most of feasible space.



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